Mathematicians' Conceptions of Active Learning and Equitable and Inclusive Teaching: A Set Theory Analysis

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Although research on active learning suggests strong connections to equitable and inclusive teaching, few studies have explored the relationship between the two concepts. For this proposal, we examined how 13 participants in an equity workshop described the relationship between active learning and equitable and inclusive teaching. We used set theory as an analytic tool to categorize their descriptions. We found that all participants saw active learning and equitable teaching as related. The two most commonly held views were that active learning and equitable teaching are related yet independent sets, or that equitable teaching is a subset of active learning. We discuss implications of these findings and pose questions for further exploration.

Keywords: Active learning, equitable and inclusive teaching, set theory

Active learning, equity, and inclusion are often-referenced terms in undergraduate mathematics education. Despite their popularity, these terms are nuanced and often difficult to define (e.g., Roos, 2019). One well-known framework for conceptualizing active learning (AL) is Laursen and Rasmussen's (2019) Four Pillars of Inquiry-Based Mathematics Education, which includes ideas about both AL and equitable and inclusive teaching (EIT). In past work (Williams et al., 2022), we used this framework to explore how various stakeholders in mathematics departments across multiple institutions conceptualized AL. We found that we were unable to categorize definitions of AL based solely on stakeholder role, and overall, stakeholders mentioned aspects of teaching and learning that related to the four pillars of inquiry-based mathematics education (students, content, teachers, and equity) in their definitions of AL. While stakeholders most frequently mentioned the actions of students in their definitions, they rarely acknowledged how equity played a role in their conceptualization of AL. This led us to wonder how members of a mathematics department perceive the relationship between AL and EIT.

For this proposal, we examined how mathematics faculty members enrolled in a year-long workshop focused on equity described the relationship between AL and EIT. We used set theory as a tool for carrying out our analysis. Specifically, if AL and EIT are viewed as two sets, then how do they intersect (or not)? Thus, our guiding research question was: How do mathematics faculty members engaged in work on active learning and equitable and inclusive teaching view the relationship between the two concepts? We provide a brief theoretical framing and background before describing how we applied set theory to understand how participants described the relationship between AL and EIT.

Theoretical Framing

Equitable and Inclusive Teaching

One definition of equity in mathematics education includes three criteria: 1) "Being unable to predict students' mathematics achievement and participation based solely upon characteristics such as race, class, ethnicity, gender, beliefs, and proficiency in the dominant language" (Gutiérrez, 2007, p. 41); 2) "Being unable to predict students' ability to analyze, reason about, and especially critique knowledge and events in the world as a result of mathematical practice, based solely upon characteristics such as race, class, ethnicity, gender, beliefs, and proficiency in the dominant language" (p. 45); and 3) "An erasure of inequities between people, mathematics, and the globe" (p. 48).

Accomplishing these three criteria requires teachers to attend to the dimensions of equitable mathematics, which lie along dominant and critical axes (Gutiérrez, 2007). The dimensions of the dominant axis are access and achievement, while the critical axis consists of identity and power. Studies on learning mathematics have frequently focused on the dominant axis by suggesting that interactive classrooms provide opportunities for students to access rich learning experiences, and by showing that student learning increases in classrooms that use collaborative, student-centered practices (Gutiérrez, 2013; Smith et al., 2021). However, creating equitable and inclusive mathematics classrooms involves thinking about how identity and power influence classroom interactions. In other words, EIT involves paying attention to how personal relationships and societal norms impact students' interactions and experiences (Gutiérrez, 2007). Power dynamics and students' identities affect students' learning experiences, and thus should be considered in mathematics classrooms that are interactive (Adiredja & Andrews-Larson, 2017).

Active Learning

There are several definitions of active learning in research literature (e.g., Williams et al., 2022; Prince, 2004; Stains et al., 2018). In their well-known metastudy, Freeman et al. (2014) provided a definition of AL that explained the difference between AL and passive lecture:

> Active learning engages students in the process of learning through activities and/or discussion in class, as opposed to passively listening to an expert. It emphasizes higher-order thinking and often involves group work (pp. 8413-8414).

Another broad definition was provided by the Conference Board of Mathematical Sciences (2016), which referred to AL as "classroom practices that engage students in activities, such as reading, writing, discussion, or problem solving, that promote-higher-order thinking" (p. 1). While these definitions provide us with some notion of what AL is, there is not a single, commonly agreed upon definition in undergraduate mathematics education. Therefore, recent studies have begun to examine the nuances and complexities of the term AL, and how it is operationalized in undergraduate mathematics classrooms (Williams et al., 2022).

Research showing that AL can lead to improved student outcomes has contributed to a consensus in the field that AL is an equitable practice (Freeman et al., 2014; Laursen, 2019); however, student experiences with AL vary (Johnson et al., 2020; Reinholz et al., 2022). There is evidence from the context of upper-division mathematics courses that when directly compared to lecture-style instruction, AL may result in greater performance disparities between women and men (Johnson, et al., 2020). Moreover, Rios (2022) found that students in introductory-level courses whose preferred math language was not English felt a decreased sense of belonging in English-dominant math classrooms as AL practices increased. These findings, coupled with evidence that AL increases achievement outcomes for students from underrepresented groups in STEM (Theobald et al., 2020), suggest that the relationship between AL and equity is complex and not fully understood.

Although research on AL suggests strong connections to EIT, such as Laursen and Rasmussen's (2019) Four Pillars framework, few studies have explored the relationship between the two concepts. Moreover, these studies tend to focus on student outcomes, rather than how the instructor's view of these concepts may influence their role in the classroom. Thus, more work is needed to understand how members of mathematics departments view the relationship between AL and EIT.

Methods

Data Collection

Data were collected from a total of 13 participants. These participants were undergraduate mathematics faculty members at a variety of institutions across the US who were participating in a year-long equity workshop as part of a larger study, which focused on AL in first-year mathematics courses. Interviews were conducted over the span of the equity workshop in September 2020, January 2021, and May 2021. Data presented in this proposal come from the final interview after the conclusion of the workshop in May 2021. As part of this interview, participants were asked to provide their definition of EIT, and discuss whether and how this definition might have changed over the previous academic year. Participants were also asked to respond to the following prompt: "Some people equate active learning with equitable and inclusive teaching. How do you see these as related, and in what ways are they different?" Participants' responses to this prompt form the data set for this proposal.

Data Analysis

To analyze the data, we used set theory to list the possible relationships between AL and EIT. If we view AL and EIT as two distinct sets (see Figure 1), we can investigate the ways in which these two sets can interact. In this context, there are five possible relationships which correspond to five diagrammatic representations. Table 1 shows the five possible relationships between AL and EIT and provides descriptions from mathematical set theory along with interpretations of how we applied this logic structure to the data in our study.

An implicit assumption that we made in our analysis is that AL and EIT exist in the same domain and that it makes sense for these sets to be intersecting. We also assumed that AL and EIT are clearly defined terms and we can thus determine what is contained in each set. We discuss these assumptions further in our Findings and Discussion sections.

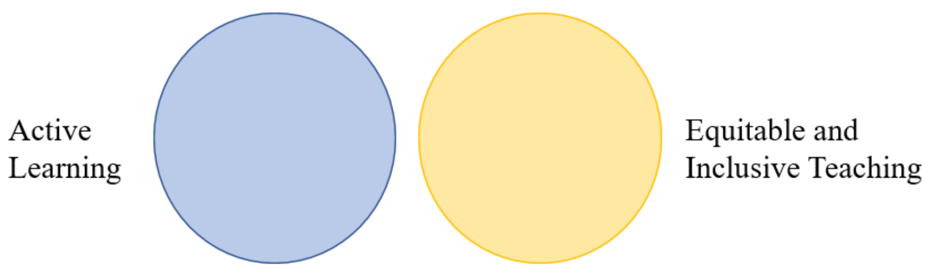


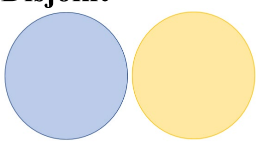
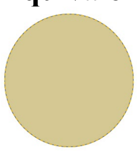
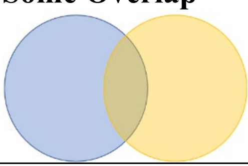
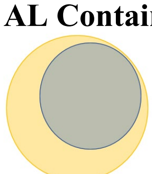
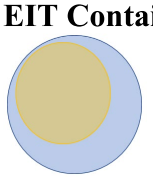
Figure 1: AL and EIT as Two Distinct Sets

After interviews were transcribed, the excerpts where participants were asked about the relationship between AL and EIT were identified and organized into a matrix for coding. Each row represented a participant and their specific response. The columns represented the different possible relationships between AL and EIT (as seen in Table 1). Our goal was to look for evidence of how the participant viewed the relationship between AL and EIT. We then decided on what relationship(s) from Table 1 the participant discussed (either saying that AL and EIT were related or were not related in the following ways) and copied text from their response to support our decision. The decision categories we used were:

- YES: if there was <u>strong</u> evidence in the participant's response that aligned with that relationship
- MAYBE: if there was <u>possible</u> evidence in the participant's response that aligned with that relationship
- NO: if there was <u>no</u> evidence in the participant's response that aligned with that relationship OR there was evidence <u>against</u> that particular relationship (directly or indirectly) in the participant's response

Every participant's response was coded independently by at least two of the authors, who then met to reconcile and discuss any differences (Creswell & Creswell, 2018).

Table 1: Set Theory Relationships between AL and EIT

Relationship between AL and EIT	Set Theory Description	Interpretation
Disjoint 	• The intersection of AL and EIT is empty ($AL \cap EIT = 0$)	Whenever AL occurs, EIT does not occur. Whenever EIT occurs, AL does not occur.
Equivalent 	• AL and EIT are equivalent sets ($AL = EIT$) • AL exists if and only if EIT exists.	Whenever AL occurs, EIT occurs. Whenever EIT occurs, AL occurs.
Some Overlap 	• The intersection of AL and EIT is not empty ($AL \cap EIT \neq 0$) AND • There exists EIT that is not AL, and there exists AL that is not EIT.	AL and EIT can occur together, and they can also occur independently.
AL Contained in EIT 	• AL implies EIT ($AL \rightarrow EIT$) • There exists EIT that is not AL. • EIT is a necessary condition for AL	Whenever AL occurs, EIT also occurs. However, EIT can occur without AL.
EIT Contained in AL 	• EIT implies AL ($EIT \rightarrow AL$) • There exists AL that is not EIT. • AL is a necessary condition for EIT	Whenever EIT occurs, AL also occurs. However, AL can occur without EIT.

Findings

Our overall findings are summarized in Table 2. We provide examples and descriptions of participants' responses in each of the set theory relationships in the following paragraphs.

Disjoint and Equivalent

All participants (13 of 13) provided strong evidence that AL and EIT were not equivalent. Similarly, almost all participants (12 of 13) provided clear evidence that they saw some relationship between AL and EIT (i.e., that these two concepts were not disjoint). For example, Lacy stated, “they’re related, but they’re not equal.” This provided evidence that Lacy saw that there was some relationship between AL and EIT, but that they were also not the same.

Emma was the only participant who potentially saw AL and EIT as disjoint concepts. In her response, she discussed the difference between AL and EIT, describing AL as a “paradigm for learning” and EIT as what the teacher is doing to create an equitable environment in the classroom:

> The definition of active learning is that students need to construct knowledge for themselves and that we can provide them with information and scaffolding and opportunities to construct it for themselves [...] this is different from equitable teaching, right? Equitable teaching is sort of making a classroom where students of any background can learn and that's different from sort of this philosophical understanding of how learning happens.

From this response, it was unclear whether Emma saw AL and EIT as disjoint (possibly in two separate universes without the same domain). One possible interpretation is that Emma saw AL from more of a student perspective (i.e., how students learn and construct knowledge in the classroom), whereas EIT is more focused on the teacher (i.e., what the teacher is doing to create an equitable environment). This led us to consider the possibility that perhaps AL and EIT could be viewed as existing in different universes (learning paradigms vs. teaching strategies) with no potential for overlap because they did not have the same domain. Therefore, we decided to code Emma’s response as a “MAYBE” under disjoint.

Table 2: Evidence of Relationship between AL and EIT

Participant	**Disjoint**	**Equivalent**	**Some Overlap**	**AL Contained in EIT**	**EIT Contained in AL**
Aadaya	NO	NO	NO	NO	YES
Bill	NO	NO	NO	NO	YES
Cassandra	NO	NO	NO	NO	YES
Crystal	NO	NO	YES	NO	NO
Collin	NO	NO	MAYBE	YES	NO
Camila	NO	NO	MAYBE	NO	MAYBE
Mark	NO	NO	YES	MAYBE	NO
Kathleen	NO	NO	YES	NO	NO
Lacy	NO	NO	MAYBE	NO	MAYBE
Emma	MAYBE	NO	MAYBE	NO	NO
Robert	NO	NO	MAYBE	NO	NO
Shea	NO	NO	NO	NO	YES
Thomas	NO	NO	MAYBE	NO	YES

Some Overlap

Most participants (9 of 13) viewed AL and EIT as related to some extent. For some, the evidence for this relationship was straightforward; Robert said, “I think they intersect, but yet they’re not equal.” For other participants (Crystal and Kathleen), their expressions supported this relationship because they claimed AL and EIT are related and provided evidence against both containment relationships. For example, Kathleen said, “I was kind of surprised by that prompt, because they definitely have an overlap, but I don't see them as being equal to each other” (evidence for some overlap); she added, “while I think that active learning benefits all students, and also helps with teaching equitably, I think that teaching equitably is also like, it includes other things as well" (evidence against EIT contained in AL), and "even though [group work] is an active learning strategy, it's not necessarily equitable unless we think about equitable practices" (evidence against AL contained in EIT). Thus, some overlap best fit Kathleen's view of the relationship between AL and EIT.

Six participants (coded as "MAYBE" for Some Overlap) gave responses indicating that they saw AL and EIT as related, but it was not clear which relationship aligned best with their explanation. One participant, Camila, explicitly used set theory in her response and said, "I haven't thought, 'which is a subset of the other,' but it could be that maybe equitable and inclusive learning is a subset of active learning. I'm not quite sure yet." Her acknowledgment of subsets shows she was considering the concepts to at least have some overlap, but then expressed doubt of a final relationship.

AL Contained in EIT

Overall, participants' responses were least aligned with the relationship that AL is contained in EIT. Only one participant, Collin, expressed strong evidence for AL existing within the set of EIT, describing AL as a "specific pedagogical strategy" within the "more ingrained [...] act of learning". He shared that

> "active learning is a specific pedagogical strategy, whereas equitable and inclusive learning is something less explicit and more ingrained in how you're doing the act of learning. [...] active learning sort of, in my mind, inherently has this idea that students are capable of learning independent of their background, so, in a sense, you are creating space for equity by allowing students to discover and have dialogue with the material."

Collin viewed AL as "inherently" equitable, implying that AL exists within the domain of EIT.

EIT Contained in AL

Seven participants' responses aligned with EIT as a subset of AL, with five providing strong evidence and two providing possible evidence for this relationship. Aadaya's response encapsulated this relationship as follows: "I think that they are related but yeah I think one would kind of be incorporated into the other." She elaborated further, stating, "So those are things you could do that maybe make students feel more included in the class when they're doing their active learning activities." In other words, in a classroom in which AL is being implemented, there are additional strategies that can be used to help students feel more included. This suggests a view that EIT is a special case of AL.

Bill described a similar viewpoint with the response, "equitable and inclusive learning … might be more purposeful." He went on to say, "there's things that you can do to be very purposeful as opposed to my current understanding of active learning as just good pedagogy, not necessarily, you know, driving in awareness of inequities." This points to a view that AL alone is not necessarily equitable or inclusive unless the instructor is "purposeful" about using equitable and inclusive teaching practices.

Discussion

Although our set theory framework allowed for the possibility of five clearly defined relationships, an overall glance at Table 2 shows that most of our participants' responses fell into two categories. The strongest evidence aligned with the relationships involving some overlap of the two concepts or containment of EIT in AL. We originally conjectured that EIT might be a larger, umbrella term that contained AL. In other words, AL as a set of teaching practices might be situated within equitable and inclusive teaching as a broader ideology (e.g., Roos, 2019). However, participants' responses primarily pointed toward the reverse containment relationship, where EIT exists within AL. This finding is consistent with studies that showed that not all AL classrooms and practices are equitable for all students (e.g., Johnson et al., 2020; Rios, 2022). We continue to explore how this finding aligns with the Four Pillars framework (Laursen & Rasmussen, 2019), where equity as a pillar could be interpreted as one element contained within the larger set of AL, or that equity is a necessary condition of AL and thus AL is contained in EIT. Furthermore, the equity workshop was situated within a larger study on AL (SEMINAL Project; see Smith et al., 2021), where instructors were first introduced to AL in mathematics instruction before connecting this concept to EIT. Thus, the primary focus on AL might have prompted participants to think about EIT within an AL context.

We also emphasize that participants shared their perceptions of the relationship between AL and EIT *at the end* of a year-long workshop on equity and equitable teaching practices. In other words, it seemed reasonable to presume that participants would have similar notions of EIT, AL, and the relationship between them. However, our data showed that participants expressed varied notions of the domains in which these terms exist (e.g., learning theories, pedagogies, specific teaching practices) and how they function in a mathematics classroom, much like Roos's (2019) findings about the role of inclusion. Our findings illustrated that even after participating in a workshop focused on equity, instructors and other mathematics faculty can still hold diverse views of the relationship between AL and EIT. This has major implications for instructors who implement these pedagogical concepts in their classes and for future professional development centered on AL and EIT. Understanding this relationship and being aware of the potential differences in stakeholders' views can inform how mathematics departments frame their collective efforts toward implementing AL and EIT, which is imperative to systemic and sustained change (Williams et al., 2022; Laursen, 2019).

In some ways, our work generated more questions than answers. For example, does having one containment view of the relationship (compared to the other) change how educators implement these practices in mathematics classrooms? Based on someone's view of the relationship, are some ideas privileged over others? Is there one "right" way of viewing the relationship between EIT and AL? Is it possible to hold multiple views of this relationship simultaneously (as responses from some of our participants suggest)? While we cannot definitively say which relationship between AL and EIT is "true" or even if one exists, our work raises questions about the implications of different views of this relationship, particularly for researchers and educators who are interested in instructional change efforts.

Finally, we acknowledge that our analysis brings to light a tension between straightforward mathematical logic and messy teaching constructs. Mathematics often involves creating well-defined categories in order to organize objects. For this analysis, we used set theory as an analytic tool to classify complex, fluid concepts within an exploratory qualitative study. As mathematicians, the structure of set theory is compelling and satisfying; however, as education researchers, we grappled with how to fit participants' responses into precisely one set relationship. Further exploration is needed to better understand the relationship between AL and EIT and the implications it has for improving undergraduate mathematics education.

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