Student Goals and Motivations in their Undergraduate Abstract Algebra Course

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Mathematics educators assign important learning outcomes to abstract algebra classes. However, it is unclear whether students share these goals or whether they have different motivations entirely. Using the framework of expectancy-value theory, we highlight preliminary results from semi-structured entrance and exit interviews with six undergraduates enrolled in abstract algebra. This report centers around the following research question: What are students' expectations, motivations, and goals for their undergraduate abstract algebra course?

Keywords: abstract algebra, motivation, goals, expectations

Introduction

Abstract algebra is a foundational course in advanced undergraduate mathematics in which students are expected to learn proof technique and abstraction alongside content-specific material (e.g. group theory, field theory) (e.g. Dubinsky et al., 1994). However, it is unclear what students' own goals and motivations are. While there have been many studies on the role of motivation in mathematics courses, there have been comparatively few studies on motivation in advanced proof-based courses like abstract algebra. This distinction is important, as mathematics educators have noted that the content and style of advanced proof-based courses differs from previous courses such as calculus (e.g. Alcock & Simpson, 2002).

We highlight preliminary results from semi-structured entrance and exit interviews with six undergraduates enrolled in abstract algebra. Our central research questions are: *What are students' expectations, motivations, and goals for their undergraduate abstract algebra course? And how do students' motivations contribute to how they approach the course?*

Theoretical Constructs

We adopt the framework of Situated Expectancy Value Theory (SEVT), (Eccles & Wigfield, 2020) to elucidate the relationship between students' expectations, motivations, and perceived task value of abstract algebra. A key assertion of Situated Expectancy Value Theory is that achievement on a task is mediated both by students' expectations for success as well as the task's perceived value (Eccles & Wigfield, 2020; Rosenzweig et al., 2019). Task value is then further separated into the constructs of attainment value, intrinsic value, utility value, and cost.

Eccles et al. (1983) define *attainment value* as the personal or identity-based importance of doing a task. *Intrinsic value* (also referred to as interest value) refers to the students' enjoyment of the task and shares similarities to the constructs of interest (Hidi & Renninger, 2006) and intrinsic motivation (Ryan & Deci, 2016). Within this study, a task is intrinsically valuable when it is enjoyable or helps procure future enjoyment. However, the concepts have different theoretical traditions and focuses. For instance, intrinsic motivation emphasizes the rationale behind an activity as opposed to what makes it valuable (Eccles, 2005). *Utility value* refers to the degree to which a task would be beneficial for a students' present or future plans. For the purpose of this study, we specify that utility value accomplish tasks for reasons other than for one's enjoyment, such as credentialing or seeking employment. Finally, *cost* refers to what an individual must give up in order to accomplish a task. We define success as the student's realization of their goals and is not necessarily based external metrics such as performance on an exam, and is thus a student-centered relative construct.

Literature Review

While motivation has been studied within math education literature (Hannula et al., 2016), and in scholarship on undergraduate STEM education (Cromley et al., 2016), we found no studies attending to motivation in proof-oriented mathematics classrooms. Existing motivation literature has largely consisted of studies drawing correlates between motivation and related constructs like self-efficacy, achievement, and interest (Cromley et al., 2016; Jones et al., 2010). There has been a recent emergence of scholarship on EVT interventions intended to increase student motivation (Rosenzweig & Wigfield, 2016). Much of the interventional scholarship has focused on utility value interventions as it is perceived to be the most easily modifiable task value (Harackiewicz et al., 2014) (i.e., the interventions have focused on increasing students' perceived utility of the tasks on which they work). While there is extensive literature on the statistical relationships between students' self-efficacy, motivation and performance in mathematics courses (for a review, see Wigfield & Eccles, 2020), there is relatively little work on how these factors operate qualitatively.

Eccles and Wigfield adopted SEVT from Eccles and colleague's Expectancy Value Theory (Eccles, 1984; Eccles et al., 1983) as part of a call to attend more closely to the situatedness of motivation. Situatedness, for Eccles and Wigfield, refers not only in the macro sense to the student's sociocultural environment but also to their interpersonal context. We believe our work provides a contribution to SEVT by situating our understanding of our participants motivation within their specific context of undergraduate abstract algebra. Accordingly, previous studies of elementary schoolers point to the domain-specificity of situated task values and academic self-concept.

Methods

Our study consisted of two semi-structured interviews with six participants on their motivation and associated study habits halfway through and at the end of the semester. Our interview protocol contained questions of the form:

1. Why do you think it might be important to study math?
2. What do you expect to get out of studying math in college?
3. What do you expect to get out of Abstract Algebra?
4. In a perfect world what would you want to get out of Abstract Algebra?

The first question asked participants for their perceived task values of studying math while the second and third question ask for participants' expectancies for both math and abstract algebra. The fourth question draws a distinction between participants' expectations and what they perceive they could optimally get out of abstract algebra.

In the second interview, we asked students to reflect on their responses in the first interview and to comment on whether they felt their expectations and/or ideal ("perfect world") goals were achieved. Students were also asked for their perceived task value of abstract algebra. We used thematic coding (Braun & Clarke, 2006) to identify key themes from the interviews.

Results

We present preliminary results from our ongoing analysis. One emergent theme is that some students desired intrinsic value, had utility value, and appreciated but did not require utility value in the form of real world applications. When asked what he expected to get out of abstract algebra, Participant 1 stated:

> It does seem like you know, at least for group theory, that that comes up a lot in things so I'll have an understanding, very foundational conceptual understanding and in a field that does have applications to these, this theoretical work. Um, hopefully, I'll come out of it. Part of the anticipation is also I don't really know where it might be useful, but hopefully, the expectation is, a useful thing will present itself to me. And also, you know, everybody I've talked to has been like, "oh, wow, Galois Theory is really awesome". […] I think it's one of those subjects which is, I'm very good at when I have an intuition of it, but sometimes something will happen I'm like, "why is that? Why does that work?" Or like "what does that work and this not work" Actually, that's the worst thing. It's not "why does this work?" But usually, 'why does this work and that doesn't'

The student remarks that he is unsure in what ways abstract algebra would be useful for him but that he expected that "a useful thing will present itself." We interpret this as the student expecting a utility value even though he does not know what it is yet. Even in the absence of a concrete utility value, the participant still anticipates the class will be intrinsically valuable to him. The expectation of intrinsic value seems sufficient motivation for Participant 1. A different participant, however, anticipated he would struggle with the course in the absence of intrinsic value. When asked what he could get out of abstract algebra under ideal circumstances, he answered:

> Yeah, I'm not too sure. I mean, I think...I don't know. I mean, I think most importantly, *for me even more than like having real life application and would be just to find something I'm interested in*, because like I said, *I have to pass the course*. And I think courses are a lot easier for me when I can find myself getting really interested in them. So for me even more important than having a real life application, just be to have an interest in it because I think it would just kind of be easy to pass the class with an interest in it, but it's kind of going in the direction where it's, it's like I don't have an interest in it. So I'm expecting to be pretty difficult for me to kind of fully understand everything and get myself to study for it. And prepare myself for exams and other things. But I, I guess, like you know, like I said, like, I mean, I don't know that anything off the top of my head in the class so far that would have a realistic you know, real life application for me, but I do think that would be very nice. That would be very cool if I could kind of get something because I think that would motivate me in itself to know that like okay, this is going to be something you have to know and have to understand versus right now I'm kind of in the like, my mind, I'm just like, "Okay, I literally just have to pass this class. And that's it.

Like Participant 1, Participant 2 desired intrinsic value in the course. While "real life application[s]" would be a plus for Participant 2, the participant stressed that, "even more important than having a real-life application [is] to have an interest in" abstract algebra. An interesting topic would make it "easy to pass the class;" without it, he would find it difficult to "get [himself] to study." We interpreted this to mean that even though Participant 2 desired utility value, it was intrinsic value (i.e., personal interest) that was crucial. It is notable that Participant 2 had a high personal utility value for the course—he required it to graduate—but thought that this would not be sufficient motivation for him to do the work to study the material. Later in the interview, Participant 2 described how the effort required to pass would be substantial, requiring frequent personal meetings with the course professor for extra help. The intrinsic motivation seemed necessary for Participant 2 to put in this effort.

Participant 3 noted utility that advanced mathematics had, but found this to be an unattractive reason to pursue the discipline.

> I study it, in part because I like it. Which is a huge motivation factor. But, you know, there's always that question of like, 'what is the real world application of what I'm doing?' and and I did struggle with that for quite a while, especially in that like transition from community college to [university]. Because [in university], a lot of the recruiting programs that you know, try to try to hire students out of the math program, they are government based or like Lockheed Martin, or, you know, all of those other programs and I don't like that very much. You know, and so a lot of times at this higher level, the application of math is based on those kinds of programs. And so, okay, I can name that as an application but I don't like that as an application.

Participant 3 later clarified that while much mathematics is used for a military focus, one can use mathematics for a “community-based focus and to do good things” and learning “flexible thinking and critical reasoning”, so mathematics could potentially have high utility. Nonetheless, Participant 3 stated that this is not why they studied mathematics:

> I think at some point, I just determined that I do math because I like it. And I enjoy it. Just like, I don't have to have a real world application for doing like arts or poetry, or whatever. So personally, I like it.

Discussion and Conclusion

Our analysis is preliminary and speculative at this stage. The data require verification with closer analysis. Our data indicate so far that for some students, attainment value is critical. Participants 1 and 3 were confident they would find attainment value and were optimistic about their ability to succeed. Participant 2, however, anticipated he would struggle in the course due to the absence of intrinsic value. Furthermore, our data suggest a connection between student motivation and the way students approach abstract algebra. Utility value appears to be the least important construct; the students included in this preliminary analysis neither expected it nor needed it. Indeed, Participant 3 thought the utility of mathematics for military use was a negative feature of the discipline. One student, Participant 2, was worried he would be unable to pass the class without finding intrinsic value and stated that he needed to go to office hours. Participant 1 and Participant 3 cited enjoyment as a reason to pursue mathematics.

Much scholarship in undergraduate math education has focused on achieving the instructor’s goals without addressing whether this aligns with student goals. Our work demonstrates that for some students, intrinsic value is the most important factor among intrinsic value, utility value, and attainment value. We suggest more research on how intrinsic value can be fostered in students and, more generally, how students can experience the joy that mathematicians experience in doing mathematics.

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