Data Scientists' Use of Embodied Ideational Mathematics in Conveying Concepts Rooted in Linear Algebra

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Embodied cognition is the theory that learning is influenced by the body through a variety of modalities. Nathan (2022) summarizes literature indicating that forms of embodiment such as body form, gesture, simulation, and materialist epistemology can support student learning. Other studies in embodied cognition focus on ways in which people's perceptions and ways of thinking about a topic can impact teaching and learning and place a high level of emphasis on ideational mathematics. Ideational mathematics is defined as an individual's perception of formal ideas (Schiralli & Sinclair, 2003; Sinclair & Gol Tabaghi, 2010). While previous studies centered on mathematicians, our work centers specifically on the emerging field of data science. Despite the significant amount of literature on embodiment in mathematics (Alibali & Nathan, 2012; Hall & Nemirovsky, 2012; Sinclair & Gol Tabaghi, 2010), there exists a gap in the literature on the impact of embodiment as it relates to data science. Thus, we wish to better understand how instructors use ideational mathematics regarding concepts they present in their classrooms. In conducting this investigation, we leverage Nathan's (2022) summary of various forms of embodiment to address our research question: In what ways do mathematicians use embodiment to convey ideational mathematics when discussing data science concepts rooted in linear algebra?

We conducted one-hour audio and videotaped interviews with two data scientists (Hunter and Elizabeth) with varying backgrounds. Hunter's area of expertise is topology, and Elizabeth's expertise is in harmonic analysis. In this poster, we present an analysis of two episodes from these interviews. The selected episodes provide evidence of Nathan's (2022) summary of the four forms of embodiment. The two data scientists primarily discussed concepts ideationally, and used representational gestures, materials, and simulation that accompanied personified verbiage. For example, Elizabeth explained an activity that she uses with students in her capstone course to teach cosine similarity within natural language processing. Her activity, which she referenced as a "Bag of Ingredients" model, uses grounded, real world examples of recipes to clarify the broader, more complex "Bag of Words" model. She related comparing ingredients in recipes to comparing words in documents of various length to discuss the comparison of common words and synonyms. Hunter integrated various tools and materials such as drawings on a whiteboard and a model of a molecule to explain principal component analysis, a key concept in the field of data science. The concepts discussed in both episodes are rooted in linear algebra, a mathematics subject which both participants identified as an area of struggle for students. The implications of the insights offered through analysis of these episodes is particularly relevant as data science programs become increasingly predominant and institutions search for the best ways to convey material.

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Motivation

- Existence of outlines of introductory data science courses (Baumer, 2015; Hicks & Irizarry, 2018)
- Studies exploring mathematicians' ideational mathematics (Sinclair & Gol Tabaghi, 2010; Soto-Johnson et al., 2016) and embodiment (Marghetis et al., 2014).
- Linear algebra in the formal, symbolic, and embodied worlds of thinking (Stewart, 2018).
- Minimal research on the teaching and learning of data science.

Research Question: In what ways do data scientists use embodiment to convey their ideational mathematics when discussing concepts rooted in linear algebra?

Principles of Teaching Data Science

- Organize around Case Studies
- Integrate Computing
- Minimize Notation
- Mimic Experience
- Demonstrate Critical Thinking

Hicks and Irizarry (2018)

Theoretical Framing

Embodied cognition: The theory that learning is influenced by the body through a variety of modalities.

Type of Embodiment	Definition	Examples
Body Form	Use of body and the environment	Tracing a circle on hand
Gesture	Spontaneous Movements of hands, arms, and other body parts	Indexical (pointing), Representational (iconic and metaphoric), Beat
Simulation	Using imagination to extend ideas	Use of MATLAB or other technologies
Materialist Epistemology	Use of materials and manipulatives	Annulus, water wheel, blocks

(Nathan, 2022)

Conceptual Mathematics

- Formal, public representations
- Examples
  - Textbook definition of Principal Component Analysis (PCA) : dimensionality reduction method which transforms a large data set into a smaller one that still contains most of the information

Ideational Mathematics

- An individual's perception of formal ideas
- Examples
  - PCA produces the best view in 2-D or 3-D space for physical, geometric intuition
  - Matrix multiplication as gobbling up a vector, throwing information away, and spitting out a vector

(Schiralli & Sinclair, 2003; Sinclair and Gol Tabaghi, 2010)

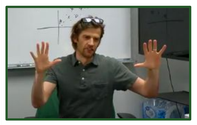
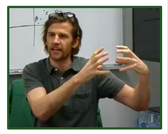
Methodology

Participants: Two **data scientists** working in a mathematics department at a research university in the Rocky Mountain area.
- Hunter - Topologist
- Elizabeth - Harmonic Analyst

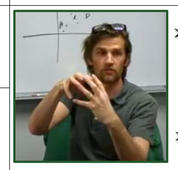
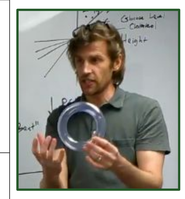
Data Collection and Analysis:
- Videotaped approximately hour-long interview with each participant.
- Open coded entire interview for embodiment (Nathan, 2022).
- One episode was selected from each interview, approximately 10 to 15 minutes in length, after the first round of coding.

Results

(1) Mathematical areas of expertise influence ideational mathematics and forms of embodiment

Participant	Embodiment Type	Transcliptions
Hunter Focused on visual & geometric representations through **body form, gesture, & materials**	**Movement, Body Form, Perception** • Used body to represent data science and mathematics as vast fields, and topology's role in each	➢ "Inside of data science, topology is a very small part, on the order of 1% of all data science" 
	Gesture - metaphoric • Topology a "pretty big area" (left) of the "vast landscape" of mathematics (right) • Described how to formalize geometric representation of data	 
	Materialist Epistemology • Whiteboard inscriptions visually represented a high-dimensional data set as: ○ Chart of numbers ○ 5D and Cartesian planes ○ Points arranged in circular formation	 ➢ "We formalize blurring one's vision by maybe growing a ball around each data point"
Elizabeth Represented physical & abstract objects through **body form, gesture & simulation**	**Movement, Body Form, Perception** • Utilized environment to fix "locations" of abstract mathematical objects	➢ Matrix multiplication "gobbles up vector" and "throws away" information" ➢ Measured by nullity 
	Gesture - metaphoric • Established "locations" of null space and column space on either side of her	
	Simulation • Described matrix multiplication as "gobbling up" and "spitting out" vectors, and "throwing away" information • Nullspace fixed to her right • Column space fixed to her left	 ➢ A vector is "spit out", measured by rank

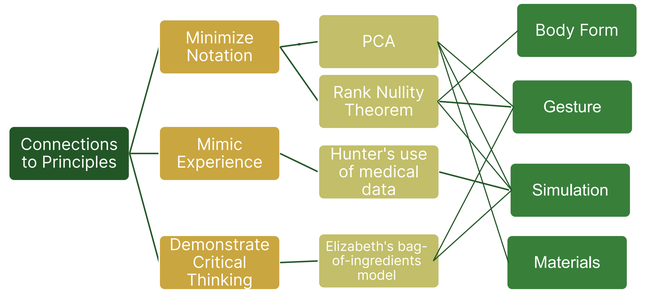
(2) Data science can be explained using relatable materials and simulations, reducing need to rely heavily on symbolism

Participant	Embodiment Type	Transcliptions
Hunter Use of relatable PCA example through **gesture, simulation, & materials**	**Gesture** - metaphoric • Uses hands to represent familiar shapes to describe data	 ➢ "Maybe the data set is arranged among a sphere, and maybe there's a good reason why the data is well-modeled by a sphere
	Simulation • Relatable PCA example: data represents medical characteristics of a doctor's patients	➢ "It can stretch and bend in fold in different ways. . .you have many different points in 24-D space" 
	Materialist Epistemology • Molecule represents 24-dimensional data set • Annulus represents the 24D data "living" in two dimensions	 ➢ "Turns out, this molecule lives on a very lower-dimensional, 2D space inside of 24 dimensions"
Elizabeth Use of relatable and immersive **simulations & gesture**	**Gesture** - metaphoric • Returns to established "location" of nullspace for song decomposition • Collided hands together representing orthogonality	 ➢ In the outputs of the decomposed songs, the "stuff that was thrown away was all the low frequency things"
	Simulations *Rank-Nullity Theorem Demo* • Decomposition of various songs based on frequencies • Used songs familiar to students *Bag-of-Ingredients Model* • Modified Bag-of-Words model using recipes and ingredients instead of random words	➢ Bag-of-ingredients didn't recognize that two different cookies were both sweet because they used different sugars, so "those two parts of the vectors were orthogonal" 

Discussion and Implications

Expertise Influence in Embodiment:
- Both participants utilized gesture,
- Hunter used **physical** materials and Elizabeth used **imagined** materials
- Need faculty with **varied expertise** to effectively **teach all students** while keeping the plethora of approaches to data science in mind (National Academies of Science, Engineering, and Medicine, 2018).

Relatable Explanations:
- Opens up "multiple pathways for students of different backgrounds to engage at levels ranging from basic to expert" (National Academies of Science, Engineering, and Medicine, 2018, p. 64)



While discussing the linear algebra concepts they thought were important in data science, the participants explained their ideational mathematics with varied embodiment reflective of their backgrounds.

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