Analyzing Student Understanding of Derivatives Through Tasks Presented Online

Enes Akbuga, Ph.D.[1], Annika Hurd[2], Zachary Benton Hurdle[3]
[1]enes.akbuga@drake.edu, Drake University, [2]Drake University, [3]Utah Valley University

Introduction

Calculus plays a crucial role in STEM (Bressoud 2021) and studies have indicated that **derivate** is a complex concept to learn (Park, 2015; Zandieh, 2000). Researchers have unpacked student cognitive processes and existing mental structures when doing mathematical tasks involving derivatives (Baker, 2000; Asiala, et al., 1997). The COVID-19 pandemic caused disruptions in mathematics courses and more instructors are using online tools like Desmos or WebAssign. We investigate research questions:

(1) To what extent are derivative tasks facilitated with prompting in an online environment helpful for student learning?

(2) What mental structures and connections exist in the student process of doing online derivative tasks?

Framework & Research Questions

APOS theory (Arnon et al., 2014; Dubinsky & McDonald, 2001) is utilized to study student understanding of derivatives. Examinations through APOS theory generally begins with the development of a **genetic decomposition** (GD) and following the GD for derivatives (Asiala et al., 1997), we explored student interpretation of derivatives. To unpack students' mental structures and processes involved, we focused on a Baker et al. (2000)'s **graphing schema** for derivatives which includes the **interval schema** and the **property schema**. Aligned with that, **the triad** (intra-, inter-, and trans) for schema development is used in the context of APOS theory.

Methodology

We conducted semi-structured clinical interviews with **five students** who had taken Calculus I at a college level. Sergio (white male) and Maxine (female of color) are incoming freshmen at state midwestern universities who had completed AP Calculus AB. Amber (white female) and Trisha (white female) completed while Vilma (female of color) was enrolled in Calculus I at a liberal arts midwestern university in the United States. All the interviews were **conducted via Zoom** while the students interacted with **Desmos**. Students described their thought processes as they worked on the two tasks related to derivatives. **The first task** pushes students to think critically about the conceptual understanding of the derivative as the instantaneous slope or slope of the tangent line while **the second task** requires creating a graph of f(x) solely based on given conditions and no equations. During the interviews, the researchers used prompts including "what is a derivative?" or asking students to evaluate f'(x) where x is an arbitrary point.

Literature Cited

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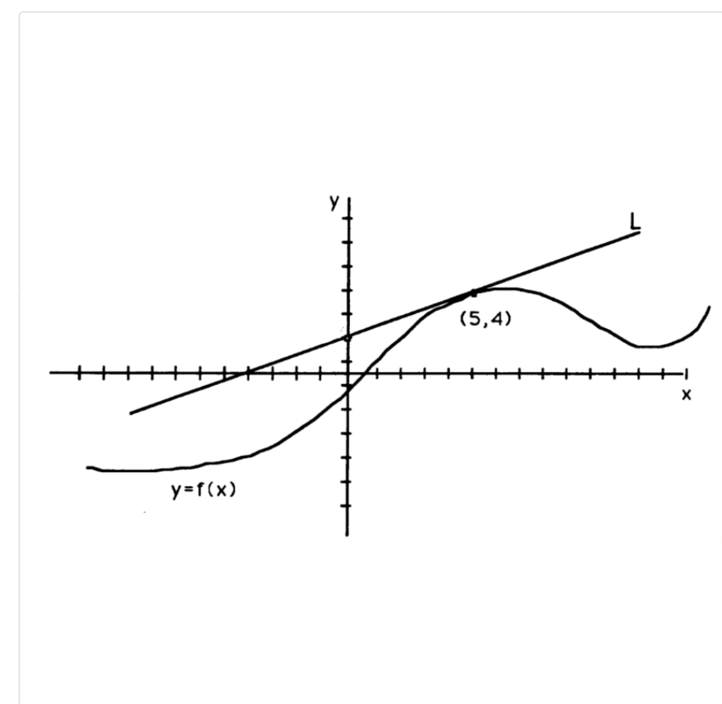
Task 1



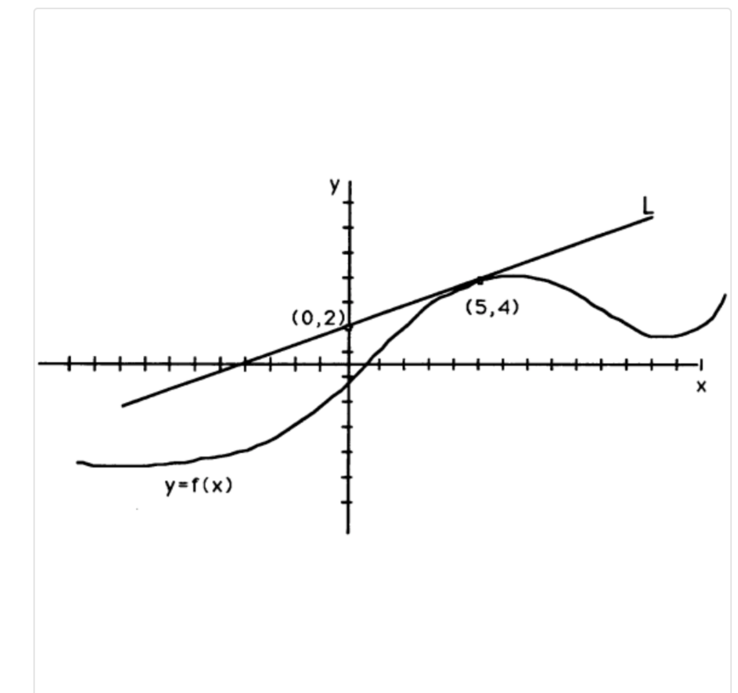
On the graph to the left, suppose that the line L is tangent to the function f at the point $(5, 4)$. Now that you found $f(5)$ How would you find $f'(5)$?

-You can show any work by sketching on the graph on the left.

-Feel free to use the text input below to provide your reasoning.

Submit

Thinking in a different way?



Could finding a slope be helpful here? What is the slope of this line?

Explain your reasoning.

How does this relate to $f'(5)$?

Explain your reasoning.

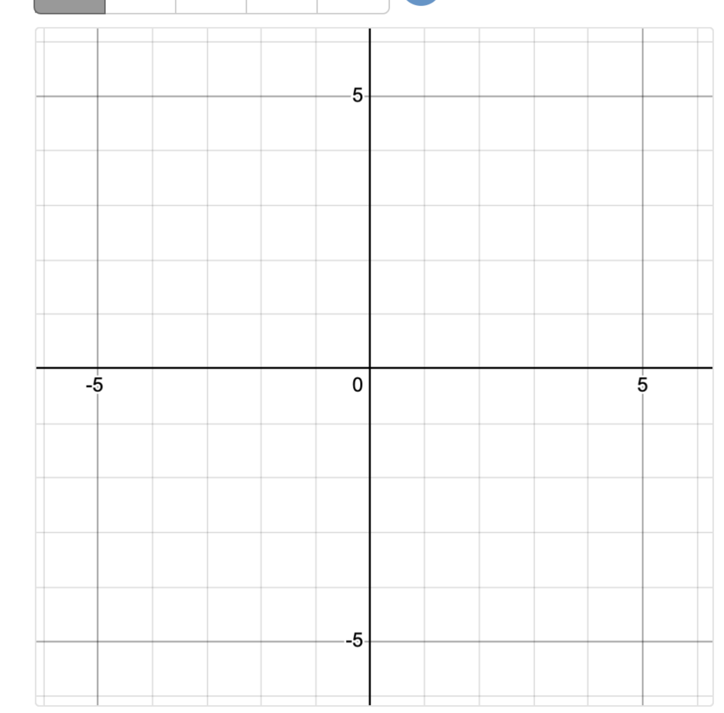
Submit

C. Graphical interpretation of the derivative at a point
- i. Overcoming the need to differentiate some formula
- ii. Coordinate with A to see f'(a) as the slope of a tangent line
- iii. Coordinate several interpretations of f'(a)

D. Graphical interpretation of the derivative as a function
- i. Seeing the derivative as the function, x → slope at (x, f(x))
- ii. Identifying f' with the line tangent at a point

E. Several coordinations to get the graph of f
- i. Graphical interpretation of f(x), for a single x
- ii. Interpretation of f'(x) for a single x as the slope
- iii. Process of x moving through an interval
  - a. monotonicity of the function and sign of the derivative
  - b. infinite slope (vertical tangent) and infinite derivative
  - c. concavity of the function and sign of the second derivative
- iv. Drawing a complete or fully representative graph

Task 2



Using the blank graph provided on the left, sketch a graph of a function f that satisfies the following conditions:

- f is continuous.

- $f(0) = 2$, $f'(-2) = f'(3) = 0$ and $lim_{x\to 0} f'(x) = \infty$

- $f'(x) > 0$ when $-4 < x < -2$ and when $-2 < x < 3$.

- $f'(x) < 0$ when $x < -4$ and when $x > 3$.

- $f''(x) < 0$ when $x < -4$ and when $-4 < x < -2$ and when $0 < x < 5$.

- $f''(x) > 0$ when $-2 < x < 0$ and when $x > 5$.

- $lim_{x\to -\infty} f(x) = \infty$ and $lim_{x\to \infty} f(x) = -2$

Submit

The nine dimensions of the Derivative Schema.		Intervals ((-4, -2, (0, 3), etc.)		
		Intra-	*Inter-*	*Trans-*
Properties (monotonicity, concavity, differentiability, etc.)	*Intra-*	Intra-intra	Intra-inter	Intra-trans
	Inter-	Inter-intra	Inter-inter	Inter-trans
	Trans-	Trans-intra	Trans-inter	Trans-trans

Above we present the first task and GD used to analyze (Asiala et. al., (1997) on the left side and second above and the triad model used to analyze that data on the table above.

Results & Discussion

All participants remarked that the tasks looked very different from the type of problems that they had done in-class or for homework in Calculus I. None of the participants had difficulty with using the online tools and Maxine appreciated constructing graphs online. Sergio notes that "...on the [usual] homework you can just find a pattern and knock'em out pretty quick…" but explains that for the novel tasks " … you can't cheat it if it's a conceptual problem, you have to actually think about it." Results suggest that students continue to have difficulties in **going from just "taking derivatives" to conceptual understanding.**

We identified components of the genetic decomposition and their critical role in student understanding of derivatives. We also found evidence of Baker's (2000) two dimensions of student understanding of derivatives (property & interval). Vilma was the only student at trans-trans dimension. We found that using certain prompts like "What is a derivative" or "what's slope elsewhere?" via online environments could also be helpful, but there is a need to investigate this further.