Sign Language Variation and Iconicity in Undergraduate Mathematics

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Two American Sign Language interpreters and two Deaf instructors were asked to sign various undergraduate mathematical terms and definitions. Various types of iconicity were identified in the signs and each term had more than one sign demonstrated for it. The participant's preference and choice for signs reflected their mathematical beliefs and prior experience. While the motivation for all participants was to accurately translate the meaning of the mathematical concepts, not all participants shared the same type of preference for sign type. One out of the four participants preferred to use signs that represent the notation used for the concept, while the other three participants preferred to use signs that showed some conceptual meaning.

Keywords: Math Education, Sign Language, Deaf Education, Linguistics, ASL, Iconicity

Background Literature

Deaf students drop of out of college at alarmingly high rates (Walter, Foster & Elliot, 1987; Walter, 2010). Most deaf undergraduates attend a university where a mainstream classroom is the only environment (Stinson, Liu, Saur & Long 1996), and thus might need to utilize interpretation services for American Sign Language. American Sign Language (ASL) is the primary language of the Deaf Community in the United States. Not all deaf people use ASL to communicate, but many students choose to utilize ASL interpreters in the classroom.

Savic and I conducted an initial qualitative study with the aim of developing an understanding for what Deaf and Hard of Hearing (DHH) students' experience is like in undergraduate mathematics (Simmons & Savic, 2018). The students acknowledged struggles related to instructor communication and struggles related to incorrect or confusing sings used by interpreters with words like "sign" and "function" (which have multiple meanings in English). If there is confusion about which signs to use for certain mathematical ideas, the investigation of ASL use in higher level mathematics may provide insight into the difficulties DHH students face in undergraduate mathematics courses.

Several researchers have used embodied cognition to analyze how students' mathematical conceptions are grounded in their bodily experiences (Núñez, 2003; Smith 2009). Núñez et al. (1999) explain that embodied cognition grounds the situatedness of knowing and learning in bodily experience and bodily actions "realized through basic embodied cognitive processes and conceptual systems" (p. 46). Embodied cognition also takes the position that mathematics does not exist independently from human understanding. "Reality is constructed by the observer, based on non-arbitrary culturally determined forms of sense-making which are ultimately grounded in bodily experience" (p. 49).

Theoretical Perspective

Meir and Tkachman define iconicity as "a relationship of resemblance or similarity between the two aspects of a sign: its form and its meaning" (2014). In essence, the opposite of iconicity is arbitrariness.

Embodied cognition has been used to study gesture in mathematical talk (Edwards 2008; 2009) and sign language use in elementary mathematics (Krause, 2017, 2018, 2019). Krause (2019) adopted theoretical constructs from Edwards work on categorizing gestures (Edwards,

2009). *Iconic-physical* reference represents physical objects or actions. For example, the sign used for the German word for simplifying uses the dominant index finger to "cross out" numbers from what looks like a numerator and denominator in the air. *Iconic-symbolic* reference represents symbolic notation or graphical representations of mathematical ideas. Krause (2019) noted that the sign for simplifying had iconic-symbolic elements because the sign represented fraction notation spatially: the index finger would be placed horizontally to represent the fraction bar. She used her own theoretical construct, *innerlanguage iconicity* to refer to signs that reference other mathematical ideas or concepts by similarity in performance of the sign. For example, the German sign for denominator "borrows" the sign for number within the performance of the sign for denominator.

The study provides evidence that sign language influences conceptualizations through what the student perceives as iconicity of signs. All three types of iconicity reference were seen to influence conceptualization of corresponding mathematical ideas. The author points out that the categorization is not "exhaustive" but can "sensitize practitioners to the choice and design of signs they use in the mathematics classroom" (p. 15).

Research Question

What types of variation and iconicity are there in the way ASL interpreters and Deaf instructors sign undergraduate mathematical terms and definitions (and what reasons do they express for these choices)?

Methods

Because of the lack of information about ASL use in undergraduate mathematics, exploratory interview methodology lends itself to provide a basis of information to conduct future studies in this area of research. Interviews held through Zoom in a variety of ways helped gather information about ASL use by interpreters, DHH students, and a Deaf instructor in undergraduate mathematics courses. The participants were able to choose their preferred mode of communication: spoken English, text-chat interview, and the use of an ASL interpreter were types of communication the participants chose to utilize. They also consented to have pictures and videos shown in reports including their faces for the purpose of retaining the meaning of the signs (as facial expression is an important element of ASL).

The four participants of this study (all residing in different states and given pseudonyms) consist of two ASL interpreters and two Deaf mathematics students/instructors. Martha and James are the two ASL-interpreter participants. James has been interpreting ASL for 27 years and is hired through an agency to interpret at a large Southwestern university. Martha has been interpreting for 35 years and is currently hired on as staff at a community college known for having a large population of deaf students.

The other two participants of the study are Deaf and have had experience learning and teaching mathematics in college: Andrew and Thomas. Andrew has a BS in mathematics from Gallaudet (a university for deaf students) where he was able to take mathematics from Deaf instructors and all his classes were taught in ASL. He is currently a PhD student and has taught 45 credits worth of mathematics in ASL. Thomas majored in four STEM fields and chose to have some of his mathematics and physics courses interpreted in ASL. He taught an undergraduate applied mathematics course in ASL and had an interpreter for his all-hearing class.

The interviews were one-hour in length and consisted of two phases: general experience discussion and term card signing. During the first half of the interview, participants were asked

questions about their general experience with ASL and mathematics, sign creation for non-standardized terms, and techniques used when signing about mathematics in the classroom. During the second phase of the interview, participants were presented with term cards that contained a mathematical term they have probably experienced (based on their response about which math courses they are familiar with) along with a definition for the term. The definitions were pulled from a variety of sources to simulate encountering a new topic in the classroom and account for course variety across institutions. All four participants were given the same instructions: to show the signs they use and have seen for the term and to sign the definition written below it.

The analysis of the data consisted of identifying each sign given for the term cards and categorizing those signs by iconicity categories: innerlanguage iconicity and iconic-symbolic iconicity. Innerlanguage iconicity has three subcategories: conceptually-linked signs, English-linked signs, and initialized signs. Iconic-symbolic reference has two subcategories: notational signs and graphical signs. While the main categories are drawn from Edwards (2009) and Krause (2019), the subcategories were created as themes found during data analysis and are defined in the results section.

Results

I have categorized the variation of sign use leaning on Krause's iconicity construct, innerlanguage iconicity (2019) and Edwards gesture iconicity construct, iconic-symbolic reference (2009). I first define the category, give an example of a sign in the category, and provide transcript of a participant's explanation or reasoning for their sign choice.

Innerlanguage Iconicity

Leaning on Krause's definition of innerlanguage iconicity, an innerlanguage iconic sign is a sign that shows reference to another sign. This type of reference can show a linkage of the referenced concepts to the concept that the sign represents. This could be in the form of borrowing the same handshape, movement, or position of the reference sign.

Conceptually-linked sign: A sign that has iconicity to another sign with the intention of showing conceptual relation between the two ideas, I refer to as a conceptual sign. Out of the nine term cards, conceptually-linked signs were given for only one of the main terms, *Derivative*. However, four of the eight variations of signs given for *Derivative* were conceptually-linked signs.

Figure 1 illustrates how Martha uses the same motion to the sign for "*change*" but uses the handshape for the letter "*d*". The use of the same motion as "*change*" is intentional in that it links the related concept of change to the concept of derivative.

Fig. 1 Martha (interpreter) is shown demonstrating the sign they use for Derivative.

In Figure 2, Thomas uses the sign "derive-from" for Derivative. This is an intentional linking of the concept to-derive-from and the concept of Derivative.

Fig. 2 Thomas (Deaf student/instructor) is shown demonstrating one of the signs he uses for derivative. He uses the sign "derive-from" for Derivative.

English-linked sign: I use the term English-linked sign to refer to signs that incorporate signs from another concept that uses the same English word but can be considered a separate concept. Figure 3 shows Andrew using the sign "*line*" for *Linear,* and Figure 4 shows him using the sign "*independent/independence*" for *Independence*.

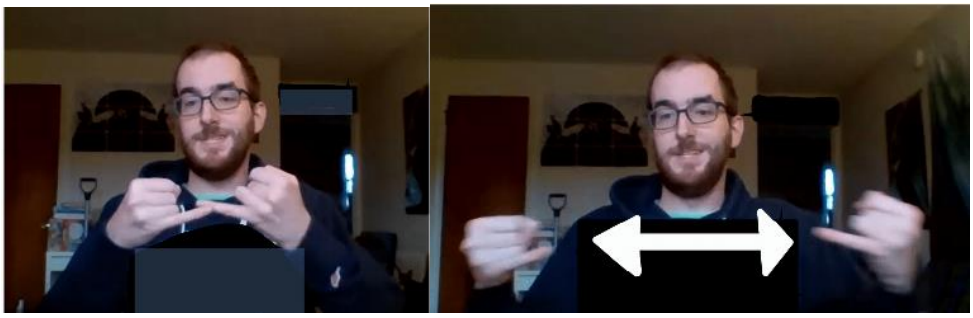

Fig. 3 Andrew (Deaf student/instructor) is shown demonstrating the first half of the sign for Linear Independence, also "line".

Fig. 4 Andrew is shown demonstrating the second half of the sign for Linear Independence, also "independence/independent".

Andrew explained that he doesn't think the sign shown above is a very good sign because the sign for line doesn't exactly mean "linear", but it is the sign that is used for the term "*linear*". It is the sign typically used for *Linear Independence* and was given by all the participants who were shown the card.

Initialized sign: The discussion of initialization came up during Martha's interview, but is seen throughout the language of ASL. If a sign is initialized, it uses the fingerspelled version of the first letter of the word. For example, the sign for "mathematics" uses the handshape for the letter "*m*" on both hands facing in as they brush across each other with the dominant hand on top. Different mathematics topics will use the same motion but with the handshape of the first letter

of the topic: algebra uses the "*a*" handshape and calculus uses the "*c*" handshape. Out of the nine term cards, an initialized sign was given by at least one participant for four of the main terms (this does not include completely fingerspelled signs which lie outside the iconicity categories).

While signing the term card for function, Martha discussed different signs she uses for *Domain* as seen in Figure 5.

Fig. 5 Martha (interpreter) is shown demonstrating two different signs she uses for Domain.

The first sign for *Domain* uses an open dominant hand moving side to side across the other arm, and the second sign uses the handshape for "*d*" with the same motion. She demonstrated similar signs for *Range* using the handshape for "*r*" and moving her hand up and down (versus side to side).

Martha described her use of initialization in the classroom for the purpose of helping the student learn the vocabulary term:

> I sometimes include more initialization if it has to do with terms that are going to come up on the test… As a technical interpreting tool, I will sometimes bring that into my interpreting to help students.

Iconic-Symbolic Reference

A sign with iconic-symbolic reference shows reference to symbolic notation or graphical representation of the signed concept. It can use motion, handshape, and position to mimic how the notation or graph looks.

Notational Sign: A notational sign has iconic reference to the symbolic notation used to reference the concept.

As illustrated in Figure 7, James (interpreter) uses the handshape for "*d*" and motions a "z" shape in the air. While interpreting for a physics student, he collaborated with other ASL users to create a unique sign for *Derivative*. He mentioned that the sign "is definitely not standard. [It's] something that the physics student, a linguist and [himself] worked on and came up with." It was influenced by the notation of . Andrew, during his interview, described a sign used by many people at RIT that resembles this sign.

Fig. 7 James (interpreter) is shown demonstrating his primary sign for Derivative.

Graphical sign: A graphical sign has iconic reference to how a mathematical idea can be represented graphically.

Fig. 8 James is shown demonstrating a sign for slope.

Both versions of the sign (seen in Figure 8) might be used for slope generally, however, the left picture, would be used to represent positive slope; the right picture would be used to represent negative slope. Notice, this is signed from the signer's perspective (rather than the audience's perspective).

Category Summary

Table 1 illustrates whether or not a sign in the given iconicity categories was presented by at least one participant for the term.

Table 1. Category breakdown of given signs for term cards.

	Innerlanguage Iconicity		Iconic-Symbolic Reference		
Terms	Conceptually-Linked	English-linked	Initialized	Notational	Graphical
Function	No	No	Yes	No	No
Limit	No	Yes	No	Yes	No
Derivative	Yes	No	Yes	Yes	No
Rate of Change	No	No	Yes	No	No
Slope	No	No	No	Yes	Yes
Span	Yes	No	No	No	No
Linear Independence	No	Yes	Yes	No	No
Concavity	No	No	No	No	Yes
Continuous Function	No	Yes	No	No	No

Discussion

There were not only differences in the way each participant signed the terms and definitions presented to them in the study, there were also different ways the signers translated the cards, often offering various ways to sign each term and definition. As ASL is not standardized at the undergraduate mathematics level, sign variation documentation can help bring awareness to alternative ways of signing mathematical ideas. Students may struggle with unfamiliar signs if they attend college in a different region than where they learned mathematics in high school.

Krause found that iconicity can influence how a student can conceptualize corresponding mathematical ideas (2019). The data from this study suggests that an interpreter, or Deaf

instructor might choose or prefer a sign as a result of their mathematical experience, not only with exposure to certain signs, but their conceptions of the mathematics as well. This inevitably will impact the sign usage and mathematical conceptions of DHH students (it may not be clear in exactly what way). This is an important result, as there is no mathematics education requirement for interpreters to interpret at the university level. It is up to the interpreter to be prepared to accurately translate the material, which possibly shouldn't be expected from interpreters who are paid by the hour of interpreting (rather than additionally being paid for time spent preparing).

While there were two categories of innerlanguage iconicity and iconic-symbolic reference, there were two major themes that seemed to underly sign choice or sign preference among the participants. Martha, Andrew, and Thomas tended to use or prefer signs that referenced the concept directly or a related mathematical concept in the performance of the sign. Typically, these signs would lie in the Conceptually-Linked and Graphical Sign categories. James, on the other hand, stated and demonstrated that he preferred signs that represented the notation of the concept (Notational Sign category). While his sign preference was different, his motivation was the same as the other three participants. They all emphasized the importance of accurately translating the mathematical concepts' meaning. James believes that a sign which bears resemblance to a related concept's sign (innerlanguage iconcity) won't always be conceptually accurate across multiple contexts. He stated that by avoiding using more conceptual signs, you can avoid attributing misconceptions to mathematical ideas. Martha believed initialization could help students with the translation of concepts from ASL back to English for test preparation. While the interpreters had more inclination of including more notation and English-related signs, their motivations were grounded in how certain signs could offer more support for the students' conceptual understanding in the classroom.

Conclusion

The ability to express mathematics in a language that can represent three-dimensional space has a great opportunity for future study. Understanding the different ways signers choose to express mathematics in ASL and why they choose to express it in a certain way can provide insight into how language plays a role in conceptualizing certain mathematical ideas. Sign Language is different to spoken language in that it provides a way to fluently discuss graphical spaces in a way that is unavailable to spoken language. Iconicity plays a big role in sign languages, and its role in undergraduate mathematics might help us understand how DHH students think about mathematics. This, in turn, could lead us to a new or deeper understanding of how all students might think about mathematics.

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