How do undergraduate engineering students engage with puzzle problems related to first-order differential equations?

Faezeh Rezvanifard
Ferdowsi University of Mashhad

Farzad Radmehr
Western Norway University of Applied Sciences
Ferdowsi University of Mashhad

This paper explores how undergraduate engineering students engage with puzzle problems related to first-order differential equations. One hundred and thirty- five undergraduate engineering students engaged with four puzzle problems related to first-order differential equations in self-selected groups of two or three students while their communications were audio recorded. The findings related to one of these problems are reported here. The results show that many students struggled with identifying how DEs could be used for modeling real-world situations, and as a consequence of that, they solved the puzzle using their physics knowledge. The findings suggest that more focus should be paid to including modeling (or puzzle) tasks in DEs courses to help engineering students engage in higher-order thinking, develop thinking skills and problem-solving strategies needed for their future careers and advanced courses.

Keywords: Puzzle-based learning, differential equations, high-order-thinking, problem-solving strategies, thinking skills.

Introduction

Many engineering students graduated with the ability of solving routine mathematical tasks, but are not successful in applying what they have learned in solving real-world problems that require critical and creative thinking (Kamsah, 2004; Falkner, Sooriamurthi, & Michalewicz, 2012). This might be due to many students are limited to textbook questions that are solved using the topics discussed in the classroom (Michalewicz & Michalewicz, 2008). Furthermore, university mathematical courses do not appeal to several engineering and mathematics first-year university students (Klymchuk, 2017).

A number of university engineering and mathematics students drop out from their study "not because the courses are too difficult but because, in their words, they 'are too dry and boring'" (Klymchuk, 2017, p. 1106). Engineering graduates need strong communication skills and team-work mindset, but some have not developed these skills and values (Kamsah, 2004; Mills & Treagust, 2003).

Puzzle-Based Learning (PzBL) has been recognized as one of the approaches that could be used to develop students' problem-solving strategies and thinking skills (e.g., critical thinking, creative thinking, and lateral thinking) (Badger, Sangwin, Ventura-Medina, & Thomas, 2012; Falkner et al., 2012; Klymchuk, 2017; Michalewicz & Michalewicz, 2008). In recent years, several studies have explored the use of PzBL in the teaching and learning of calculus (Badger et al., 2012; Klymchuk & Staples, 2013). However, a literature search revealed no study exploring the impact of using PzBL in the teaching and learning of Differential Equations (DEs).

DEs play an important role in engineering and mathematics (Maat & Zakaria, 2011; Rowland, 2006). DEs are frequently used in different disciplines, including engineering, to model real-world problems and understanding real-world phenomena (Arslan, 2010). This study seeks to fill the gap by exploring how undergraduate engineering students engage in solving puzzle problems related to first-order DEs. The research question sought to answer in this study is *How do undergraduate engineering students engage with puzzle problems related to first-order differential equations?*

Puzzle-Based Learning

PzBL refers to engaging students with puzzle problems to enhance students' problem-solving strategies and thinking skills (Michalewicz & Michalewicz, 2008). It could be considered as a sub-set of Problem-Based Learning (Thomas, Badger, Ventura-Medina, Sangwin, 2013). PzBL could help students develop their competency in solving real-world problems in their future careers (Falkner et al., 2010, 2012). Puzzle problems can activate higher-order thinking as typically they are non-routine problems that require a great deal of *analyzing* the given information, and students often need to *create* a new strategy for solving those problems (Radmehr & Vos, 2020). PzBL helps students learn mathematical concepts meaningfully and increase students' motivation to learn mathematics (Klymchuk, 2017). PzBL impacts the classroom environment by making it more entertaining and increasing students' curiosity and participation in classroom discussions (Klymchuk, 2017).

There are three types of puzzle problems: *Sophism*, *paradox,* and *puzzle* (Klymchuk, 2017). A sophism can be defined as an "intentionally invalid reasoning that looks formally correct, but in fact, contains a subtle mistake or flaw" (Klymchuk, 2017, p. 1106). A paradox is a "surprising, unexpected, counter-intuitive statement that looks invalid but in fact is true" (Klymchuk 2017, p. 1106). Puzzle refer to "non-standard, non-routine, unstructured question presented in an entertaining way" (Klymchuk, 2017, p. 1106). Puzzles are similar to mathematical modeling tasks, which is an important part of engineering and mathematics education (Klymchuk, 2017).

A task is called a puzzle problem if it has most of the four following criteria: *Generality*, *simplicity*, *Eureka factor*, and *entertaining* (Badger et al., 2012; Michalewicz & Michalewicz, 2008). In relation to the *generality*, puzzle-problems "should explain some universal mathematical problem-solving principles" (Michalewicz & Michalewicz, 2008, p. 3). Regarding *simplicity*, the puzzle-problems should be easy to state and remember (Michalewicz & Michalewicz, 2008). To meet the *Eureka factor* criterion, the solution of the puzzle-problems should not be immediately intuitive and finding the correct path to solving them may take time and energy. When students find the correct approach to solve a puzzle problem after spending much effort, the feeling that they have at that moment is called *Eureka* moment- Martin Gardner's Aha! - (Michalewicz & Michalewicz, 2008). Finally, regarding *entertaining*, puzzle-problems should be enjoyable activities that encourage students to keep looking for a correct solution for puzzle problems (Michalewicz & Michalewicz, 2008). There is no need for all the puzzle problems to have the above criteria (Michalewicz & Michalewicz, 2008). The best puzzle problems for teaching mathematics are those that have both a conventional and a lateral thinking solution (Thomas et al., 2013). Lateral thinking and creative thinking are closely related (De Bono, 1990), and therefore engaging in solving puzzle problems could improving students' creative thinking (Klymchuk, 2017; Thomas et al., 2013).

Differential Equations

DEs play an important role in engineering and mathematics students (Maat & Zakaria, 2011; Rowland, 2006). DEs are used in many disciplines, including engineering, to model real-world situations (Arslan, 2010). It is important that engineering students develop a strong knowledge of how different problems could be modelled using DEs and how different types of DEs could be solved (Kwon, 2002).

There are three typical approaches for solving DEs: algebraic (analytic), qualitative (graphical), and numerical (Rasmussen, 2001). Analytical methods are techniques for finding the symbolic form of DEs' solutions, while qualitative and numerical methods focus on finding the graphical and numerical forms of DEs' solutions, typically with the help of technology (Rasmussen, 2001).

The majority of students in traditional introductory DEs courses are inclined to use analytical methods for finding solutions to DEs (Arslan, 2010; Keene, 2007). However, these methods are not effective in solving all types of DEs (Rasmussen, 2001). Furthermore, previous studies pointed out students' competencies in using graphical and numerical methods for solving DEs are limited (Camacho-Machín, & Guerrero-Ortiz, 2015; Czocher 2017). Many students struggle with solving DEs tasks, and the relationship between DEs and their solution is not meaningful for them (Arslan, 2010; Rasmussen, 2001) due to having an instrumental understanding of DEs (Arslan, 2010; Rasmussen, 2001).

Engaging students with mathematical modelling tasks is an effective way to improve students understanding of how DEs can be used to solve real-world problems; develop students' problem-solving skills; and increase students' motivation for learning DEs (Czocher, 2017). Furthermore, modelling tasks could help students develop their competencies in communicating their ideas, educate them as independent learners, and prepare them for their future careers (Ärlebäck, Doerr, & O'Neil, 2013). However, many students in traditional DE courses encounter difficulties finding and formulating a meaningful DE when engaging with modelling tasks (Rowland, 2006). These difficulties could be related to students' understanding of DEs being procedural instead of conceptual due to heavy emphasis on analytical techniques in traditional courses for solving DEs (Arslan, 2010; Rasmussen, 2001).

Research method

In this study, a sequential mixed method study (Yin, 2014) was designed to explore how undergraduate engineering students engage in solving puzzle problems related to first-order DEs. The participants were 135 undergraduate engineering students from a public university in the east of Iran. Four puzzle-problem tasks (one sophism, one paradox, and two puzzles) were designed and were validated by two senior lecturers of DEs. Then, the tasks were piloted with eleven mathematics students. After minor changes to the wording of the tasks, the participants engaged in solving these tasks in self-selected groups of two or three students and audio-recorded their communication. In this paper, we report how students engaged with one of the puzzles. The data were analyzed using an inductive content analysis approach (Vaismoradi, Turunen, & Bondas, 2013) as previous studies have not explored how students engage with such tasks in DEs.

The puzzle problem

This task explores how students engage in constructing a suitable DE using their knowledge of mathematics and physics. Regarding the entertaining criteria of puzzle problems, integrating physics with DEs increases students' motivation to learn DEs (Rowland & Jovanoski, 2004). Additionally, many students are familiar with such physics problems in their high school education or when passing an introductory physics course in their first-year university study; however, they might not have the experience of how DEs could be used for solving such problems.

At 9 o'clock this morning, Asghar and Farzad flew from Tabriz to Ankara on a private jet with the weight of 265 kg. On the way to Ankara, on Van Lake, at the height of 6,000 meters from the lake's surface, the jet's engines failed. Consequently, the jet crashed vertically at a speed of 300 km/h. Eight seconds after the engines failed, Farzad pressed the emergency exit button. After pressing the emergency exit button, it usually takes 4 seconds for the pilot and the co-pilot to be ejected. Do Asghar and Farzad succeed in rescuing themselves? (Consider the air resistance coefficient as $5\ kg/km$ *and the total weight of Asghar and Farzad to be 135 kg).*

Regarding simplicity, this task is not very long and relatively easy to understand and cannot be shortened if one wants to keep its context. By considering air resistance and gravity force, students could solve this task and reach the eureka factor when they find out whether Farzad and Asghar were rescued. For instance, they could use the following DE $v' = g - \left(\frac{k}{m}\right) v$ (where *v* is the velocity of the jet, g is the acceleration of gravity, *k* is air resistance coefficient, and *m* is the weight of the jet), and find the distance between the jet and the ground after 12 seconds. Regarding generality, students by engaging in this task, could develop further how they could use their knowledge of DEs to solve physics problems related to the falling object and, more generally, how DEs could be used for calculating acceleration and velocity.

Results

The majority of students' communications (forty-one groups (87%)) were related to modeling the situation using their physics knowledge, and they could not find how DEs could be used for solving this task: "what is the relationship between physics problem and DEs. I have not seen any physics problem that could be solved using DEs". The remaining six groups engaged in modelling the situation using DEs, and one group was successful in doing so.

Of the forty-one groups, twenty groups (42%) realized that this task is similar to physic questions that can be solved using the Newton's second law (i.e., $\sum F = ma$); however, twenty-one groups (45%) used other physical formulas, such as $v^2 - v_0^2 = 2a\ \Delta x$ and $x = \frac{1}{2}\ gt^2 + v_0 t$.

Sixteen groups out of the twenty groups (34%) identified that the sum of the jet's vertical external forces is the product of the constant mass and the vertical acceleration of the jet. Furthermore, they realized that the resistance of the air influences the jet's motion in the opposite direction, and it is proportional to the velocity of the jet**.**

Nine groups (19%) first solved the task without considering the air resistance, and they found that Farzad and Asghar would succeed in rescuing themselves. Thus, they concluded that when they were rescued without air resistance, subsequently, Farzad and Asghar would be rescued by taking into account the air resistance because the air resistance impacts the motion, in the opposite direction.

The other seven groups (15%) considered air resistance from the beginning of their calculations (Figure 1)**.** Thirteen groups (28%) had difficulties considering the real-world factors, i.e., air resistance and its impact on velocity and acceleration of the motion. A sample response was: "We do not consider air resistance in physics; I do not know how to take it into account. So, let us solve it without air resistance".

$$\sum F = ma \longrightarrow \text{forces in the direction of motion} - \text{forces in the opposite direction of motion} = ma$$

$$mg - kv = ma \Longrightarrow (400 \times 9{,}8) - \left(5 \times 300\frac{km}{h}\right) = 400a$$

$$\Longrightarrow 3920 - \left(15 \times 300 \times 10^3\frac{m}{h}\right) = 400a \Longrightarrow a = -5{,}984$$

$$h = \frac{-1}{2}at^2 + v_0t + h_0 \Longrightarrow h = \frac{-1}{2}(-5{,}984)(12^2) + 300 \times 10^5 \times 12$$

$$= 430{,}848 + 36 \times 10^7 \simeq 3600$$

Asghar and Farzad have been rescued because $6000 - 3000 > 0$.

Figure 1. A sample response to the task without using DEs

As highlighted earlier, six groups (13%) constructed a mathematical model using DEs. These groups used the Newton's second law to construct the following equation, $m\,g - k\,v = ma$. Then, by considering $a = \frac{dv}{dt}$, they formed a DE model and used the separable technique for solving the DE. They also calculated c using the velocity of the jet when the engines failed ($t = 0$). Afterwards, these groups computed the jet's velocity 12 seconds after the engines failed and used $v^2 - v_0^2 = 2a\Delta x$ to find Δx, in order to compare it with 6000m, the height of the jet when the engines failed. They concluded Farzad and Asghar were rescued after 12 seconds because the jet has not crashed in the lake by that time**.** An excerpt from a group is provided below:

Mahdi (The names used in this paper are pseudonyms): We had this type of question in physics, but we did not consider the air resistance. I think we should use the Newton's second law.

Kian: yes, I agree, but we should write the equation based on the Newton's second law, and consider air resistance...

Mahdi: [after 5 minutes] We have $mg - kv = ma$ from the Newton's second law. If we consider $\frac{dv}{dt} = a$, we can calculate velocity in each moment.

Parsa: It seems reasonable. Let us solve the equation. [they solved the problem together]

$$\mathrm{mg} - \mathrm{kv} = \mathrm{m}\frac{\mathrm{dv}}{\mathrm{dt}} \Longrightarrow 4000 - 5\mathrm{g} = 400\frac{\mathrm{dv}}{\mathrm{dt}} \Longrightarrow 800 - \mathrm{v} = 80\,\mathrm{v}' \Longrightarrow 10 - \frac{\mathrm{v}}{80} = \mathrm{v}' \Longrightarrow$$

$$\mathrm{v}' = \frac{800-\mathrm{v}}{80} \Longrightarrow \frac{800-\mathrm{v}}{80} = \frac{\mathrm{dv}}{\mathrm{dt}} \Longrightarrow \frac{\mathrm{dt}}{80} = \frac{\mathrm{dv}}{800-\mathrm{v}} \Longrightarrow \frac{1}{80}\mathrm{t} = -\ln(800 - \mathrm{y}) + \mathrm{c}$$

By considering the conditions: $\mathrm{t} = 0\,, \mathrm{v} = 300$.

$$-\ln 500 + \mathrm{c} = 0 \Longrightarrow \mathrm{c} = \ln 500 = 6.2$$

$$\frac{1}{80}\mathrm{t} = -\ln(800 - \mathrm{v}) + 6.2 \overset{\mathrm{t}=12}{\Longrightarrow} \mathrm{v} = 363$$

$$\mathrm{v}^2 - \mathrm{v}_0^2 = 2\mathrm{g}\,\Delta\mathrm{x} \Longrightarrow 364^2 - 300^2 = 20\,\Delta\mathrm{x} \Longrightarrow \Delta\mathrm{x} = 2088.45 < 6000]$$

Kian: so, they were rescued.

The remaining five groups (11%) were not successful in solving the DE they had formed. Three groups (6%) had difficulty in calculating $\int \frac{dv}{mg-kv}$. For instance, one group considered $u = mg - kv$, and calculated the integral as follows: $\int \frac{du}{-ku} = \ln u = \ln\frac{mg-kv}{mg}$. The fourth group found the general solution correctly; however, they made a mistake in calculating the particular solution when substituting the initial values ($t = 0$, $v \simeq 83.3\ {}^{m}/_{s}$). The fifth group found the following DE $v'(t) + \frac{5}{m}v = g$, and then represented it based on the derivative of y, i.e., $y''(t) + \frac{5}{m}y'(t) = g$. However, they were not able to solve it. They highlighted that after solving the DE and founding a general solution for $y(t),\ y(12)$ should be calculated, and if it is less than 6000, Farzad and Asghar were rescued.

Discussion and conclusion

This paper explores how students engaged in solving a puzzle related to first-order DEs. The findings indicate that many students struggled with identifying how DEs could be used for solving the puzzle and consequently used their physics knowledge to solve the puzzle. Puzzles are similar to modeling tasks (Klymchuk, 2017), and therefore, our findings could be compared with those studies that explored how university students engage with modeling tasks.

The findings are in line with previous studies (Britton, New, Sharma, & Yardley, 2005; Crouch & Haines, 2004; Klymchuck, 2010; Rowland, 2006) that reported many students are not capable of using their mathematical knowledge for finding an appropriate model for solving real-world problems. For instance, Klymchuck (2010) highlighted that "many students have difficulties in translating the word problem into the mathematical formula and then deciding which mathematics they should use" (p. 82). This might be because students did not have enough experience in solving modeling problems in the context of DEs (Crouch & Haines, 2004). As Czocher (2017) highlighted, in many traditional DEs courses, lecturers incline to emphasize more analytical techniques in their teaching.

Our anecdotal observations also showed that many lecturers in their introductory DEs courses do not focus enough on solving real-world problems and modeling tasks using DEs due to time restriction and intense curriculum. If lecturers discuss modelling (or puzzle) tasks in lectures more often and include such tasks in students' assignments and possibly exams, students would have more opportunities and motivation to develop their problem-solving strategies needed for successfully engaging with mathematical modeling or puzzle tasks, and could also help students develop their conceptual understanding of DEs (e.g., Arslan, 2010; Rasmussen, 2001).

Engaging students with puzzle problems could create opportunities for students to engage in productive struggle, one of the main elements of powerful mathematical classrooms (Schoenfeld, 2014). Additionally, one of the main purposes of mathematical service courses such as DE is preparing students for the mathematics they will encounter in their future courses or careers (Czocher, 2017; Falkner et al., 2012). Challenge students to use their mathematical knowledge to solve modelling (or puzzle) problems could help them develop problem-solving strategies and thinking skills needed for future careers (Falkner et al., 2012) or advanced courses.

References

Ärlebäck, J. B., Doerr, H. M., & O'Neil, A. H. (2013). A modeling perspective on interpreting rates of change in context. *Mathematical Thinking and Learning,15*(4), 314–336.

Arslan, S. (2010). Traditional instruction of differential equations and conceptual learning. Teaching Mathematics *and its Applications: An International Journal of the IMA, 29*(2), 94–107.

Badger, M., Sangwin, C.J., Ventura-Medina, E. and Thomas, C.R. (2012). *A guide to puzzle-based learning in STEM subjects*. University of Birmingham. http://www.heacademy.ac.uk/resources/detail/disciplines/stem/guide_to_PzBL.

Britton, S., New, P. B., Sharma, M. D., & Yardley, D. (2005). A case study of the transfer of mathematics skills by university students. *International Journal of Mathematical Education in Science and Technology, 36*(1), 1-13. http://dx.doi.org/10.1080/00207390412331271401.

Camacho-Machín, M., & Guerrero-Ortiz, C. (2015). Identifying and exploring relationships between contextual situations and ordinary differential equations. *International Journal of Mathematical Education in Science and Technology, 46*(8), 1077–1095.

Crouch, R, & Haines, C. (2004). Mathematical modelling: Transitions between the real world and the mathematical word. *International Journal of Mathematics Education in Science and Technology, 35*(2), 197-206.

Czocher, J. A. (2017). How can emphasizing mathematical modeling principles benefit students in traditionally taught differential equations course? *The Journal of Mathematical Behavior, 45,* 78-94.

Falkner, N., Sooriamurthi, R., & Michalewicz, Z. (2012). Teaching puzzle-based learning: development of basic concepts. *Teaching Mathematics and Computer Science, 10*(1), 183–204.

Kamsah, M. Z. (2004). Developing Generic Skills in Classroom Environment: Engineering Student's Perspective. *In Conference on Engineering Education (CEE 2004)* (pp.14–15).

Keene, K. A. (2007). A characterization of dynamic reasoning: Reasoning with time as parameter. *The Journal of Mathematical Behavior, 26*(3), 230–246.

Klymchuk, S. (2017). Puzzle-based learning in engineering mathematics: students' attitudes. *International Journal of Mathematical Education in Science and Technology, 48*(7), 1106–1119.

Klymchuk, S., & Staples, S. G. (2013). *Paradoxes and sophisms in calculus* (Vol.45). MAA.

Kwon, O. N., Rasmussen, C., & Allen, K. (2005). Students' retention of mathematical knowledge and skills in differential equations. *School Science and Mathematics, 105*(5), 227–239.

Maat, S. M, & Zakaria, E. (2011). Exploring Students' Understanding of Ordinery Differential Equations Using Computer Algebraic System (CAS). *Turkish Online Journal of Educational Technology-TOJET, 10*(3), 123-128.

Michalewicz, Z., & Michalewicz, M. (2008). Puzzle-based learning. Hybrid Publisher.

Mills, L. e., & Treagust, D. F. (2003). Engineering education- Is problem- based or project-based learning the answer. Australasian journal of engineering education, 3(2), 2-16.

Radmehr, F., & Vos, P. (2020). Issues and challenges for 21st century assessment in mathematics education. In L. Leite, E. Oldham, A. S. Afonso, F. Viseu, L. Dourado, H. Martinho (Eds.) *Science and mathematics education for 21st century citizens: Challenges and ways forwards* (pp. 437–462). New York: Nova Science Publishers.

Rasmussen, C. L. (2001). New directions in differential equations: A framework for interpreting students' understandings and difficulties. *The Journal of Mathematical Behavior*, *20*(1), 55–87.

Rowland, D. R. (2006). Student difficulties with units in differential equations in modelling contexts. *International Journal of Mathematical Education in Science and Technology*, *37*(5), 553–558.

Rowland, D.R., & Jovanoski, Z. (2004). Student interpretations of the terms in first-order ordinary differential equations in modelling contexts. *International Journal of Mathematical Education in Science and Technology, 35*(4), 503–516.

Schoenfeld, A. H. (2014). What makes for powerful classrooms, and how can we support teachers in creating them? A story of research and practice, productively intertwined. *Educational researcher, 43*(8), 404–412.

Thomas, C., Badger, M., Ventura-Medina, E., & Sangwin, C. (2013). Puzzle-based learning of mathematics in engineering. *Engineering Education, 8*(1), 122-134.

Vaismordi,M., Turunen, H., & Bondas, T. (2013). Content analysis and thematic analysis: Implications for conducting a qualitive descriptive study. *Nursing health science, 15*(3), 398-405.

Yin, R. K. (2014). *Case study research: Design and methods.* United State of America: Sage publication Inc.