A Student Perspective on Excellence in Undergraduate-Mathematics Instruction

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The question of what high-quality undergraduate-mathematics instruction entails has been examined in prior studies mainly by directly analyzing lecturers' teaching practices. The current exploratory study aims to provide a complementary angle by inquiring into a student perspective on this topic. Based on student-centered data consisting of interviews and questionnaires, the findings point towards 8 characteristics of "excellent lecturers" as perceived by undergraduate students. These provide student-driven support for some of the prior lecture-centered research claims, though also highlight the importance students attribute to the affective dimension in teaching, which has not yet received sufficient research attention.

Keywords: Teaching practices, High-quality instruction, Students' perceptions, Affect

> *"During the studies of an average student, he or she will encounter different types of lecturers [...], yet there are only a few lecturers that the mere mentioning of their name puts them at a different league above the rest. They are considered mythological warriors who stand side by side with students in their struggle to succeed at the [university] despite all difficulties." (a student quote from an article on an excellent undergraduate mathematics lecturer, published in a student-run university magazine; Reuven, 2014)*

Recent years have shown a growth of interest in undergraduate mathematics education within the research community (Biza, Giraldo, Hochmuth, Khakbaz, & Rasmussen, 2016; Nardi & Winsløw, 2018). In particular, one topic that has received research attention regards the characterization of quality undergraduate-mathematics instruction, aiming to provide a practical contribution to the improvement of undergraduate teaching (e.g., Biza et al., 2016; Speer, Smith, & Horvath, 2010). Studies addressing what it means to be a good undergraduate lecturer have mainly done so through empirical data stemming from lecture observations and interviews with lecturers, as well as by theorizing what constitutes high-quality teaching (e.g., Bergsten, 2007; Schoenfeld, Thomas, & Barton, 2016; Wood & Su, 2017). However, there seem to be relatively few studies that inquired into a *student perspective* when attempting to characterize the teaching practices of good lecturers. As illustrated by the quote above, students may share strong opinions when asked to evaluate their lecturers, where "only a few lecturers" would be considered by students as "a different league above the rest". Accordingly, the current study aims to provide another "brick" in the growing corpus of studies on undergraduate teaching by focusing on the characteristics and traits students attribute to those lecturers they regard as "excellent teachers".

Characterizing High-Quality Undergraduate-Mathematics Instruction

Previous studies have offered various (yet often overlapping) frameworks for characterizing what it means to be an excellent lecturer. In one line of studies addressing this topic, lecture observations served as the main source for data analysis. For example, Bergsten's (2007) study, examining a Calculus lecture, provided a list of criteria accounting for high-quality instruction: *information delivery*, *connections*, *rigor-intuition*, *algebraic-imagistic modes*, *gestures*, *socio-mathematical norms*, *mathematical mind*, *inspiration*, *personalization*, and *general criteria*. Most of these criteria focus on the mathematics-related dimensions of teaching, such as making

connections across different topics (the criterion "connections") or balancing between formal and informal mathematical registers (e.g., "rigor-intuition"). However, several of the criteria also emphasize the importance of lecturers serving as role models for students, providing motivation and support (e.g., "inspiration" and "personalization"). In another study, Movshovitz-Hadar and Hazzan (2004) analyzed an Algebra lecture and offered six principles characterizing an excellent lecturer's rhetoric. For example, Principle 1 suggests first providing a "top-down" description of the main ideas and only then a "bottom-up" construction of the details; and Principle 6 suggests explicitly distinguishing between important and less important ideas of the lecture.

A more theoretical approach was taken in another line of studies by conceptualizing which dimensions could be used to describe, analyze, and improve undergraduate-mathematics instruction. For example, through a review of prior research, Speer et al. (2010) suggested different cognitively-oriented dimensions for analyzing undergraduate teaching practices, among them: *selecting and sequencing content*; *motivating specific content*; *allocating time within lessons*; and *asking questions, using wait time, and reacting to student responses*. Also Pritchard (2010) utilized a literature review to offer a framework that accounts for effective lectures based on three interrelated dimensions: *communicating* mathematical ideas, which requires clarity and flexibility in the delivery while responding to instantaneous student feedback; *modeling* mathematical reasoning while highlighting different heuristic approaches to problem solving; and *motivating* students by expressing enthusiasm and cultivating a "learning community".

However, as noted earlier, relatively few studies have explicitly included a student perspective when examining and analyzing undergraduate-mathematics instruction. Within these, a particularly interesting case is presented in Lew, Fukawa-Connelly, Mejia-Ramos, and Weber's (2016) study, which involved not only an observation of a lesson by a reputable lecturer, but also subsequent interviews with students in order to examine their understanding of the lesson. Though the direct analysis of the lecture identified characteristics of quality instruction, the interviews surprisingly revealed that the students did not comprehend the main ideas of the lecture. One of the explanations provided by Lew et al. regarded the students' prioritization of the written formal content on the board, whereas the main ideas were expressed by the lecturer only orally. Another study incorporating a student perspective on undergraduate teaching is offered by Marmur and Koichu (2018), who identified 8 types of lesson-events the students found conducive to their learning, characterized in relation to their lecturers' instructional acts (see also the notion of *key memorable events*; Marmur, 2017, 2019).

The importance of incorporating a student perspective in research on effective teaching practices has been repeatedly acknowledged in studies situated at the school level, recognizing the mathematics classroom as a place of meaning-negotiation between teachers *and* students (e.g., Kane & Maw, 2005; Kaur, 2008). Taking these ideas to research at the tertiary level, I suggest that an inquiry into a student perspective may offer lecturers an opportunity to reflect on, and possibly re-evaluate, their teaching practices according to a hierarchy of which of these teaching practices students value most. Accordingly, the current study focuses on students' perceptions of undergraduate teaching by addressing the following research question: *How do university students characterize high-quality undergraduate-mathematics instruction?*

Method

The data for this exploratory study are comprised of semi-structured interviews with 5 students and questionnaires filled by another 18 students at a highly-ranked research university in Israel. Due to this paper's length constraints, while nonetheless wishing to discuss the findings in detail, in what follows, I focus only on the data from the interviews. The interviewed students

were in their second (n=1) and third (n=4) year towards a BSc in electrical engineering. The choice of interviewing students in a later stage of their degree was based on the assumption that students who had already experienced different mathematical courses with various lecturers would be able to provide a more informed opinion on what high-quality instruction entails.

The participants were asked to prepare for the interviews by contemplating how they would characterize an excellent lecturer while thinking about specific examples in advance. The notes they made in preparation for the interview were collected as part of the data. During the interviews, the students were first asked to describe their general experience of mathematics lectures thus far and were subsequently guided in sharing detailed accounts and characteristics of good (and for contrasting purposes, also "bad") lecturers they had studied with. The duration of the interviews ranged from 30 to 50 minutes, and they were audio-recorded and transcribed.

For the data analysis, I employed an inductive approach, where categories emerged from dominant themes through a process of iterative engagement with the data (Yin, 2011). The themes were based on the students' perceptions of what it means to be an "excellent lecturer", a "bad lecturer", and perhaps most interestingly, nuanced distinctions the students made between lecturers who were "good" and those who were "excellent". While acknowledging the study's limitation in making inferences from a small number of students, I suggest that the following analysis considerations offer support to the validity of the findings: (a) each of the themes characterizing excellent lecturers was identified based on statements by all interviewed students, with the exception of one at most; (b) statements in support of the identified themes were expressed *repeatedly* by the students throughout their interviews; (c) the significance of the themes to the students was further supported by a strong affective dimension in the students' statements, inferred either directly through expressions of emotion (e.g., "something I really *loved* about these lecturers") or implicitly through the use of intensifying adverbs (e.g., "Dr. B is *very very very very* didactic"); (d) additional support in the other collected data (i.e., questionnaires and students' notes); and (e) examining and contextualizing the themes in relation to the research literature, thus aiming to offer *analytic generalizability* rather than *sample-to-population generalizability* (Firestone, 1993), as further addressed in the Discussion section.

Findings

The analysis stage revealed 8 themes characterizing high-quality undergraduate-mathematics instruction according to a student perspective. These are described and exemplified below.

Adopting a Student Perspective

A repetitive theme characterizing excellent lecturers was that they were able to see the mathematical material and its possible difficulties from a student perspective. The ability to "get into the student's head" was described by the participants as a feature that distinguishes between lecturers who are good and those who are excellent. For example, Student D said:

> "I had lecturers that I can say were good but not excellent. I think the difference maybe goes beyond the teaching of the material. It's that if a student asks a question, then they know how to answer it in a satisfactory way, as if they entered the student's head."

According to the interviews, a lecturer that adopts a student perspective can support students' learning in both cognitive and affective terms. From a cognitive perspective, the teaching should address and incorporate the ways in which students construct mathematical meaning and develop their understanding. From an affective perspective, the acknowledgment of the student perspective also provides emotional support by creating an environment in which the learning process is a journey the students do not have to walk alone; as for example shared by Student A:

> "You feel that he [Dr. B] is walking with you hand in hand […]. Like he's coming from inside you, from within the audience [of students]. It's like he's learning the material with you all over again, while also showing it to you along the way."

Performativity

"A good lecture acts a site for 'awe and wonder'." (Rodd, 2003, p. 16). The data here support this claim and suggest that an excellent lecturer demonstrates performative abilities, including attributes such as charisma, sense of humor, and leadership qualities. It is a person who has a passion for the subject, and moreover, is able to connect students to this passion as well. For example, Student E described the difference between an excellent and a good lecturer based on their performative abilities in keeping students involved and interested:

> "Part of the reason there's such a consensus about Dr. B [being an excellent lecturer] is because of his theatrical abilities and his charisma. There's no doubt this is part of it. […] There are lecturers whose complete lack of charisma is the reason why students will fall asleep. Last semester I studied Probability with Dr. T, who, as I said before, is a good lecturer. But people would simply lose him on a regular basis because he's so monotonous. In the end, there's a tidy notebook […] because that's how he teaches the course. But on the other hand, if you were sitting like a mannequin all semester long, it's probably going to be very hard for you to prepare for the exam."

Furthermore, the interviews suggest that this teaching characteristic also supports students' emotional well-being and raises their motivation to understand the taught mathematics. For example, Student E stated that Dr. O's "humor helped lift people's spirits and pick them back up. […] When you take something heavy and difficult and deflate it with a stab of humor, then suddenly things are not scary anymore." And Student A shared the following:

> "I think that a trait that characterizes a good lecturer is leadership, […] it's a matter of who excites students, who succeeds in making them feel connected. […] And then, even if students don't understand something, then they will ask. […] It means that I'll care enough to come and ask the lecturer about those things I didn't understand."

Making Connections and Providing Directionality

This characteristic of excellent lecturers regards making mathematical connections in the following ways: (a) connections between past, present, and future ideas of the course; and (b) connections between concepts learned in the course and their utility both within and outside the mathematical domain. For example, student C discussed the importance of connections both at the local level of the material and at the global level of how ideas combine to a "bigger picture":

> "There are lecturers that at the beginning of each lecture say a sentence or two about what we did last time and then make the connection to what we'll do with this now – which is something I really loved about these lecturers. […] I really felt that lecturers who truly taught well showed how to look at everything in the context of the big picture. This was really helpful. And it was also very interesting seeing how different things connect."

Moreover, the participants reported that when such connections are made, this gives them a sense of directionality in their studies, which addresses both their cognitive and affective needs:

> "It's about clearing the fog for students, so they would know a bit more where this [material] is heading towards, as well as why. […] We are curious people, so the 'why' is

important to us. When we don't know, then there's this fog along the path […]. So this freaks us out. You don't know what's going to happen. It drives you crazy." [Student A]

Positive Attitude Towards Students

"A good lecturer is a lecturer who cares" [Student D]. The importance of lecturers caring about their students' understanding was a repeating theme: "Caring. This is a trait that really influences the atmosphere in class, in the sense of how much you'll feel comfortable asking questions. […] And also caring in the sense that they [lecturers] will really speak with you until they see you understand" [Student C]. According to the interviews, this could be achieved if the lecturer were to use inviting language that encourages students to ask questions:

> "I understand that the bigger the group is, the harder it is to give personal attention. […] But at least the lecturer should give the students the feeling and confidence of 'you can come to me, you can approach me, we'll help you'." [Student A]

Furthermore, the data suggest that lecturers' attitude may have a substantial impact on students – "The attitude of the lecturer can really make you love or hate the course" [Student A]. Whereas lecturers who demonstrate a positive attitude towards students may raise students' involvement, lecturers who behave dismissively towards students may cause students to dislike the course – even when taught by knowledgeable, organized lecturers; as shared by Student B:

> "Dr. E is really organized and knows the material excellently. But if you ask a question in her lesson, then you'll receive a very cold shower, you'll be humiliated. […] Like, serious humiliation. If she had done it to me, I would have been really insulted. […] Like, yelling at you to go over the material again in front of a class of several-hundred students […]. On the other hand, Dr. M, her attitude is amazing. She answers questions calmly."

Keeping It Simple

The mathematics that undergraduate students need to learn and deal with is often complex and challenging. However, based on the interviews, an excellent lecturer is able to thoroughly *simplify* the material. One of the methods mentioned by the students was using "extremely simple examples" to demonstrate definitions and proofs. The students emphasized that it is the deliberate *choice* of which examples are presented that can make even complicated material become simple, understandable, and clear. For example, Student A shared the following:

> "Dr. Z, what Dr. Z does, she brings the material to amazing simplicity. […] She explains it by bringing it to such a simple level. She simplifies everything and gives the most basic examples in the world, that not understanding them […] is just impossible."

Other ways of simplifying the material included the use of illustrations, gestures, simple language, and daily situations to explain the material. For example, Student B stated:

> "He [Dr. B] really brings every theorem to the level of a person on the street, like even they could understand what he's talking about. [...] He takes a complex idea and really simplifies it. [...] I'll give you an example. When he explained the definition of a limit of a function, I think everyone understood it. […] He used words that are very intuitive. You can imagine it, it's words that create an image in your head. He uses a lot of words like that, it's very very easy to follow, very easy to connect. […] So suddenly even this scary definition feels much more understandable."

Calm Time Management

While the data suggest students acknowledge the time constraints on lecturers, they nonetheless characterize excellent lecturers as lecturers who are able to manage their time in a

calm manner. This as opposed to lecturers who seem to be "running against the clock" in their teaching in order to cover the material. As articulated, for example, by Student D: "What I less liked about this lecturer was that she was super stressed and super quick – if you blinked for a second, then you lost her." Also Student E shared a similar stance: "If a lecturer is speeding through the material, this means that something in the planning of the schedule is not right."

It should be noted, however, that according to the students, *calm* time management does not mean *no* time management. For example, Student E claimed that a good lecturer knows when to insert pauses, which "can even be just a few seconds long" – allowing the students to finish copying from the board or "the time needed for an understanding to kick in". Student E explained this could prevent students from feeling "lost" and "entering a whirlpool of frustration". Another repeating claim was that good time management not only regards allocating time for answering students' questions calmly, but also knowing when lecturers need to say "no" to additional questions and instead continue in the lesson; as, for example, shared by Student C:

> "It's really nice when a lecturer answers everyone's questions and does it really patiently. But many times I encountered the situation where the lecturer answered too many questions, and then we simply didn't have the time to learn what we were supposed to."

Meticulous Preparation and Presentation

Another characteristic of excellent lecturers is an extremely orderly and thorough presentation of the material, which also stems from being rigorously prepared for each lesson. For example, consider the following excerpt by Student A, who described excellent lecturers as having high pedagogical awareness in both planning and executing the lesson:

> "Everything is very thorough and everything is very orderly, and you see that there's an orderly lesson plan that they thought about in depth: what comes before what, where should the example go, where should the theorem go? I think that at this level, Dr. B is very very very very didactic, and like, really, he's at the highest level I have seen."

According to the interviews, other aspects in lecturers' meticulous preparation and presentation of the material include a very deliberate choice of examples in the preparation stage, as well as making sure to not only explain the material orally, but also write all their explanations on the board (cf. Lew et al., 2016). As, for example, claimed by Student C: "I think it's really good when lecturers write down *everything* they explain. Especially if these are important things. That they really write in an orderly manner on the board, so we could look at it afterwards."

Creating a Hierarchy of Importance in the Mathematical Material

While the students acknowledged that attention to detail is crucial in mathematics, they also wished to be guided on which of these details they should concentrate and focus on first, before attempting to make sense of others. The characteristic "creating a hierarchy" thus regards lecturers helping students distinguish between more important and less important aspects of the material. As illustrated by Student A, not knowing what is vital to comprehend and what is less crucial to understand may result in students experiencing anxiety and stress:

> "When you don't know what to focus on, then you don't know what is important [and] you don't know what's not important. […] When you don't know what to focus on, everything puts you into some kind of panic mode, and this is what makes it so difficult."

According to the interviews, one way in which this hierarchy can be created is by lecturers repeating and summarizing only the most important and relevant information at the end of each lesson or topic. Another way regards lecturers first providing students with the general structure

of the proof and only subsequently filling in the details (see also "structural proof"; Leron, 1983). Student B described this teaching practice, as well as its affordances for student learning:

> "Every good instructor does this. […] They create […] a fragmentation of the solution into blocks [and explain] 'this is what I'll to do [in each block]'. […] This is good for a lot of reasons. […] If you lost some part of the proof, a block in proof, you can say, 'well, I don't understand what the lecturer is doing right now, I'll wait for the next block, I'll accept the previous block as a given'. This might sound a bit funny, but in the end, it's the difference between understanding and not understanding anything."

Discussion

The motivation for this study stemmed from a gap in the research on undergraduate-mathematics instruction, which has not sufficiently incorporated a student perspective when inquiring into this topic. Addressing this gap in this initial, exploratory study, 8 characteristics of high-quality instruction were identified based on student-centered data, offering the following contribution to undergraduate-mathematics research.

First, I suggest that the findings herein provide important initial support to prior research on undergraduate teaching by adding a student perspective to the discussion of the topic. For example, the characteristic "Positive attitude towards students" could be regarded as a student validation to one of Jaworski's (2002) elements in her teaching triad, *sensitivity to students* (regarding their affective needs); "Calm time management" resonates with Speer et al.'s (2010) theoretical discussion on *allocating time within lessons*; "Making connections" across topics, a notion that has been widely discussed in prior lecturer-focused studies (e.g., Bergsten, 2007; Yan, Marmur, & Zazkis, 2020), seems to indeed motivate students; and "Creating a hierarchy of importance" supports prior characterizations of effective teaching, such as Leron's (1983) notion of *structural proof* and Movshovitz-Hadar and Hazzan's (2004) claim that good instruction requires distinguishing between important and less important ideas of the lecture.

Secondly, and perhaps more importantly, the student perspective on teaching as reflected here might challenge us, as a research community, to widen our focus of attention towards the affective dimension that is inherent to any teaching-learning interaction. As can be observed in the findings above, there is a strong affective dimension in the students' descriptions of which teaching qualities they found significant to their learning experience in lectures. This affective dimension was expressed not only regarding characteristics that explicitly attend to students' affective needs (e.g., "Positive attitude towards students"), but also regarding characteristics that are often viewed through a purely cognitive lens – for example, the characteristic "Making connections", which when not met, "drives you crazy", as argued by Student A. Furthermore, the affective dimension in the students' interviews was often inseparable from the cognitive one, supporting a non-dualistic approach to affect and cognition in mathematics education (e.g., Roth & Radford, 2011); for example, consider the characteristic "Adopting a student perspective", which tends to both students' cognition *and* affect. While we may find some discussion of affect-related characteristics of high-quality undergraduate teaching in prior studies, this dimension is often presented as secondary and subordinate to the cognitively-oriented teaching characteristics (e.g., Movshovitz-Hadar & Hazzan, 2004). Accordingly, I suggest not only that further empirical research into the student perspective is needed, but also that the data analysis of such empirical studies would benefit from further development of theoretical frameworks that highlight the affective dimension of undergraduate-mathematics teaching and learning.

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