An Instrumental Approach Towards Graduate Teaching Instructors' Use of Applets

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This study investigated how mathematics instructors' instructional goals for teaching constant rate of change (CROC) influences their perception and use of applets in mathematics instruction. This report presents results from a clinical interview with graduate teaching instructors (GTIs) to illustrate the degree to which their mathematical meanings for teaching (MMT) impact their image for how an applet should be used and how the GTIs' use of the applet influences their MMT. I use instrumentation theory as a lens for explaining the relationship that develops between an instructor and particular features of the applet. I conclude with a discussion of how GTIs MMT for constate rate of change impacts their intended use of an applet's feature and how they imagine they can use an applet to advance students' mathematical understanding of CROC.

Keywords: Applets, Instrumentation Theory, Teachers Mathematical Meanings, Technology

Introduction and Literature Review

As technology advances, digital technologies such as graphing software and applets are being created and used in elementary, secondary, and undergraduate mathematics education (Eason and Heath, 2004) to support students' mathematical learning. Applets are interactive computer-based objects that enable students to visualize attributes of specific mathematical ideas (Gadanidis, Gadanidis, & Schindler, 2003). The dynamic abilities of an applet can promote discussions that focus on quantities and their relationship within a problem context as they vary in tandem (Moore, 2009).

Although applets are frequently included in mathematics curricula as resources to support students' learning and performance, few instructors are using applets in their teaching (e.g., Engelbrecht & Harding, 2005; Daher, 2009). Teachers' pedagogical beliefs, mathematical knowledge, classroom management, and comfort with applets have been identified to influence their perception of applets and how they incorporate applets into their instruction (Gadanidis et al., 2003). When solving mathematical problems, some teachers perceive applets as tools for visually representing quantities within a problem context, thus potentially supporting students in constructing an appropriate image of the problem context (Daher, 2009). However, a teacher's perspective on teaching and her view of what is involved in learning a particular idea or making sense of a problem context might influence whether or not they use the applet.

Over the years, researchers (Bowers, Bezuk, & Aguilar, 2011; Moore, 2009; Guy, 2020) have demonstrated the benefits of applets as tools for supporting student mathematical learning when utilized as a *didactic object* (Thompson, 2002). An object (e.g., a visual aid, a graph, or an applet) is not considered didactic until the teacher or researcher views the object as a tool for producing reflective classroom discussions focused on a particular theme or way of thinking (Thompson, 2002). Moore (2009) revealed that an applet implemented as a didactic object could enable and sustain discussions that focus students' imagery on coordinating changes in the vertical distances' values above the horizontal diameter and the changes in arc-lengths' values as they varied in tandem. An important feature of his implementation of the applet was his decision to use the enactment of the applet's dynamic features as a way to verify and/or modify students' predictions of how the quantities' values may vary in the context of trigonometry. His implementation of having students anticipate allowed them the opportunity to imagine how the quantities varied and reflect on their conjectures relative to their conception of the dynamic contextual situation. Moore's use of the applet to support students' conception of trigonometric function was driven by his *mathematical meanings for teaching* angle measure.

Thompson (2016) explained that a teacher's *mathematical meanings for* teaching (MMT) consist of the teacher's images of the mathematics they teach and the intended meanings that they hope their students to have. A teacher's mathematical meanings are described as productive when the meanings she holds are productive for students' mathematical learning, in the long run, if they were to hold them as well (Bylerley & Thompson, 2017). Thompson, Philipp, Thompson, & Boyd (1994) stated that teachers' images of the mathematics they teach influences how they implement curricula in their classroom(s). These researchers characterized and defined these images as teachers' orientations towards mathematics teaching. Thompson et al. (1994) claimed that a teacher with a *conceptual orientation* tends to focus their students' attention away from the thoughtless application of procedures and provide opportunities for students to conceptualize quantities in problem contexts and explain the rationale for claims. In contrast, a teacher whose actions are driven by their image of mathematics as the application of calculations and procedures for producing numerical results ("solving task to just get an answer") is described to possess a *calculational orientation* towards teaching mathematics.

Theoretical Perspective

Prior research on the instrumental approach has proven to be fruitful for investigating how technology can be used and what shapes its use during an interaction between the learner and a digital tool. Instrumentation literature (e.g., Vérillon & Rabardel, 1995; Artigue, 2002; Thomas & Holton, 2003; Drijvers, Kieran, Mariotti, ... & Meagher, 2009; Drijvers, 2019) has focused on describing the interaction between the student and digital tool. In this study, I use instrumentation theory as a lens to examine the initial interactions between a teacher and an applet that they are viewing for the first time.

An instrumental approach consists of one's transition in viewing an object as an artifact (e.g., a tool) to an instrument. When an object is viewed as an instrument, there exists a bilateral relationship for the use of the tool, in which, on the one hand, the user (and their mathematical meanings) shapes the techniques for using the tool, and on the other hand, the tool shapes and transforms the user's mathematical practice (Drijvers, 2019). An *artifact* is described as an object that is used as a tool[1]. Whereas an *instrument* is a conceptualization that entails the uses of an artifact with a specific goal in mind (e.g., GTIs' uses of an applet with their teaching goals in mind). The use of an artifact as an instrument entails the researcher's (or teacher's) *schemes* and *techniques* for how they imagine using the artifact. I define a *scheme* as the organizations of mental activity (or actions) that includes a set of conditions that trigger the scheme and an expectation of the results of having applied the scheme (Piaget, 1971). *Human activity* (e.g., a set of gestures, ways of thinking, and actions both physical and mental) by an individual in order to perform a given task (Trouche, 2005) is referred to as the individual's use of the applet. Thus, as a GTI experiences one or several features of an applet, I (the observer) will speak of it as a *conceptualized use of the applet* or an *instrumented technique*.

The process of building an instrument from an artifact is called *instrumental genesis* (see Figure 1). This process consists of two interconnected components: One component is

[1] An applet described in instrumentation theory is already assumed as a tool (Russell, 1997) and not as an object that one must come to view as a tool. The latter view of an applet as a tool stems from a radical constructivist (Glasersfeld, 1985) stance that I adopt.

instrumentalization, which is a process where the user (e.g., GTI) constructs an image of what the artifact is designed for and how it should be used. For an applet, this entails the activity of selecting and discovering relevant features (e.g., clicking on boxes or moving sliders) and unplanned attempts to transform certain functions of the applets (e.g., the interface) with the goal of modifying and personalizing the applet to one's liking. Instrumentalization can be described as the process of a GTI differentiating between their MMT for a mathematical idea and their image of the purpose of the applet's capabilities and features.

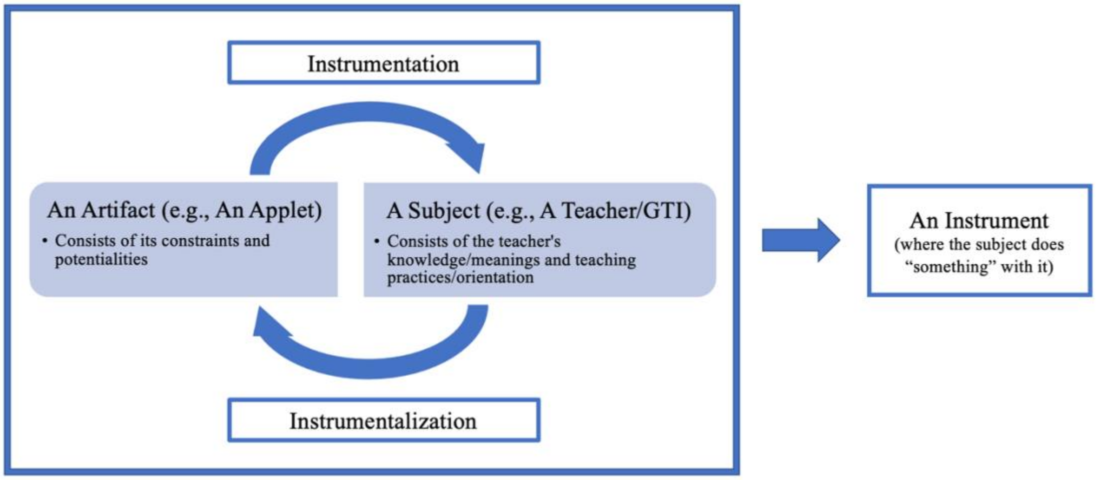

Figure 1:The transition from an artifact to an instrument through instrumental genesis.

The second component is *instrumentation*. This is the process through which the constraints and potentialities of an artifact shape the subject and how they may use the tool during their instruction. During this process, one goes through the emergence and evolution of schemes while performing tasks of a given type. Defouad (2000) described this process in two phases, which I will explicate in relation to this study. The first phase, which happens at the beginning, entails the GTI attempting to find a balance between their former teaching practices and their imagery for conveying mathematical ideas with the newfound capabilities from the tool. The second phase is where a state of equilibrium is established (rather the subject is aware of it or not) in the use of the applet features and capabilities that align with the GTIs instructional goals and what they perceive the applet to be useful for. Hence, this report investigates instructors' conception of the applet's usefulness in supporting students' understanding of constant rate of change and answer the following questions:

- *How do GTIs' mathematical meanings for teaching influence the ways in which they spontaneously incorporate applets in preparation for instruction?*
- *In what ways are GTIs' instructional goals for the concept of constant rate of change aligned with how they imagine incorporating applets in their teaching?*

Methodology

This study involved two graduate teaching instructors, Ebony and Armando, at a large public university in the southwest United States. The GTIs teaching experience of the *Pathways Precalculus* (Carlson, Oehrtman, & Moore, 2018) curriculum ranged from 2-3 years, with each completing one year of a professional development seminar called TUME.[2] (Teaching

[2] This seminar included mathematical tasks designed and sequenced to support the GTAs in constructing productive meanings for key mathematical ideas in the precalculus curriculum.

Undergraduate Mathematics Education). The GTIs participated in one clinical interview (Clement, 2000) where the researcher asked the GTIs to complete two tasks designed to reveal their thinking and meanings for CROC. Following the task, I presented the feature of an applet designed to support student understanding of the idea of CROC to the GTIs and asked them to think aloud as they interacted with the applet. This consisted of open-ended questions that focused on how the GTIs intended to use the applet to support their instructional goals for the task & CROC. Interviews lasted 1.5 hours and were recorded using Zoom Video Communication technology (2020) to capture facial gestures, audio, and written responses with transcripts.

The interviews were analyzed in two phases following an open coding approach (Strauss & Corbin, 1990). The first phase entailed analyzing GTIs' mathematical meanings for teaching CROC. The second phase entailed analyzing teachers' interactions with the applets through the lens of the instrumental approach. This consisted of identifying moments in which GTIs leveraged their MMT and instructional goals for CROC when using specific features of the applet.

A candle is 8.5 inches tall when it has been burning for 6 hours at a rate of ¼ in. per hour. How tall was the candle before it was lit?

Figure 2: The Candle Burning task (Carlson et al., 2018).

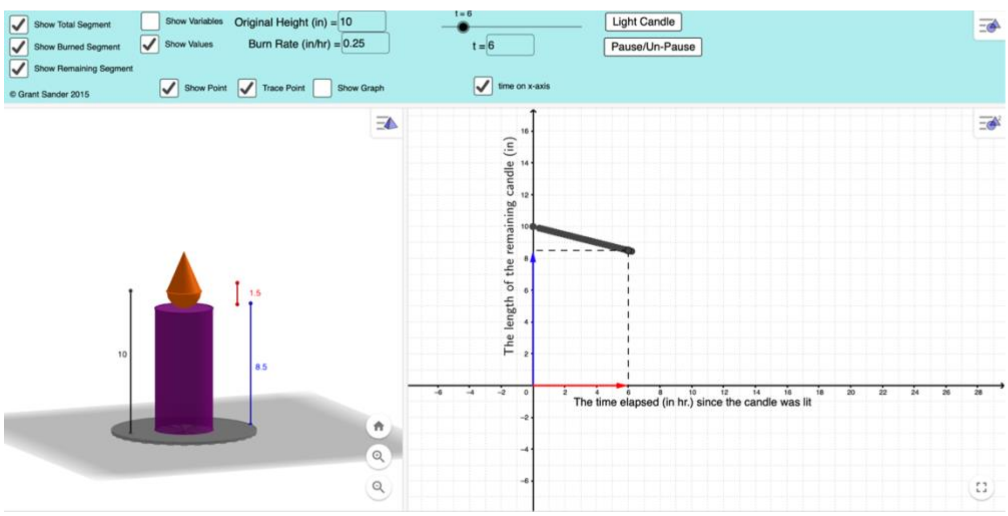

Figure 3: A screenshot of the candle burning applet (Rendered in GeoGebra by Sander, 2015)

Results

I share the following results from the GTIs' interview with Task 1 (Figure 2) and the Candle Burning Applet (CB-Applet, see Figure 3) from the Pathways Precalculus course materials. I first highlight the instructors expressed meanings for CROC and their image of the ideas they hope to convey to students. Next, I illustrate the GTIs orientation to teaching mathematics by describing their image of the pedagogical task (e.g., the goal it serves) and the questions they intended to ask when using the applet during discussions with their students. Lastly, I share how the GTIs expressed meanings and orientation toward teaching CROC impacted their image of how they intend to use the applet and how this use is aligned with their instructional goals.

GTIs' Meanings and Orientation for Teaching CROC

Ebony began the interview by reading the prompt (Task 1) aloud and explicated how she would determine the original length of the candle (see Figure 4, left panel). Next, I asked Ebony how she would use this task in her class. She indicated that she would lead a discussion aimed at getting her students to attend to the quantities and how they are related. She explained that she wanted to support the students in conceptualizing an appropriate image of the problem context. She later described this conversation as being centered around a key way of thinking that she later defined as *quantitative reasoning* (Thompson, 2008). Next, she further explained that she would focus students' attention on the variations in the remaining length of the candle and burnt lengths of the candle with respect to time elapsed. She also described *covariational reasoning* (Thompson & Carlson, 2017) as another way of thinking that she imagined students must be engaged in to support their understanding of CROC as she intended. She stated that with the help of prior conversations about the quantities, their relationships, and the construction of tables of the covarying values that students meaning of CROC (in the context of the problem) may express "the change in the length of the candle burned is always negative 1/4th times as large as any change in time." During her interaction with the applet, Ebony expressed the goal that her students understand CROC as entailing a constant ratio between two covarying quantities. This evidence suggests that Ebony's image of this task included a conversation centered around CROC as a way for describing how the changes in one quantity's values change relative to the changes in another quantity's values in the context of a burning candle.

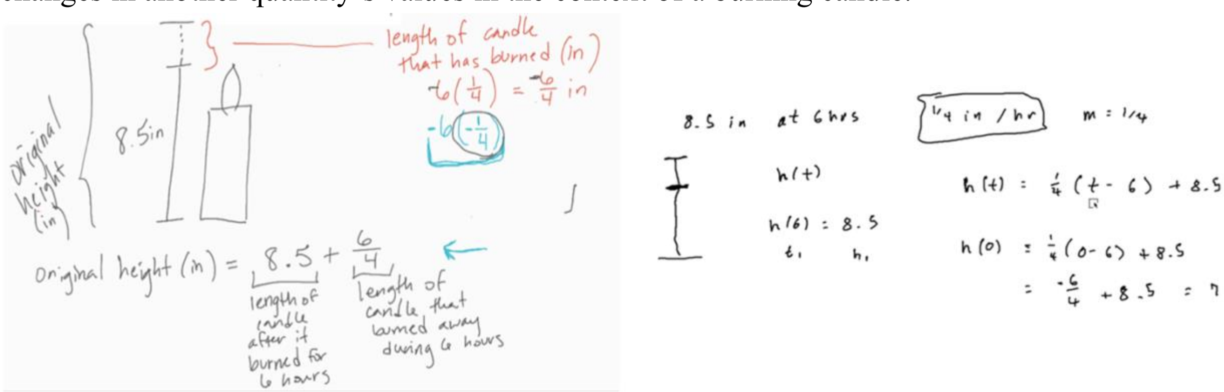

Figure 4: Ebony (left panel) and Armando's (right panel) written work for Task 1.

In contrast to Ebony, Armando read the prompt aloud and began to write a function formula to determine the initial height of the candle in terms of time (see Figure 4 right panel). Armando stated that if he presented this problem to his student, he would prefer to go right into generating a formula to represent the situation. But for the prescribed curriculum, it was important to attend to quantities as stated in the TUME seminar because this was key in supporting students in constructing meaningful formulas. For this task, Armando stated that the intended meaning for CROC that he imagines his students to develop is that the expression $\Delta h/\Delta t$ is a ratio that represents a single number "no matter what two points you pick" (referring to his drawn graphical representation of CROC where there exists a CROC between the remaining length, *h*, and time, *t*). He further expressed that by showing his students the definition (see Figure 4 right panel), the goal of the task is to support students in understanding the general case, which can be represented by constructing the point-slope formula. In addition, he explained that CROC "is a way for characterizing linear relationships and as far as the intuition and sort of the pictorial reasoning and physical reasoning goes… You know, I hope that they (the students) think that the constant rate of change is all about linear relationships. All about lines." This statement suggests that Armando's image of CROC entailed a computational and geometric representation of a line that is comprised of a linear relationship.

The GTIs' Use of the Applet

As Ebony first interacted with the CB-Applet, she voiced her appreciation for the interface of the applet as the image of the candle burning in the left panel aligned with her goal of supporting students' image of the problem context. Ebony clicked on each of the boxes and moved the sliders to discover the capabilities of the applet. She mentioned that the variations of the blue and red vectors on the axes correspond with the line segments on the candle (both on the interface of the applet and in her constructed image). Next, Ebony clicked on the following boxes in this particular order: *ShowPoint*, *ShowTrace*, then *ShowGraph*, then proceeded to describe her intended use of these functions. She verbalized that the use of these three features supported her goal of engaging her students in coordinating the changes in the two quantities values as they vary together (see Excerpt 1).

Excerpt 1

Ebony: Okay, so that's nice, that it doesn't just immediately show the graph. So, I like this because I imagine using these three features [*referring to ShowPoint, ShowTrace and ShowGraph*] to help students...to help the graph emerge in students' thinking. So we could, like, really have a conversation about covarying quantities where I would like to turn on this point [*only leaves the ShowPoint box checked*] and have conversations about like, okay, so like. What does it... why is that, what does it mean for that point to be on the y-axis [Ebon*y had moved the t-slider to 0 with the original height to 10 in*.]? Now let's imagine that the candle has burned for one hour, where do you imagine you're going to see that point on the graph and we could actually like, show them [*moves t-slider to 1 hour and circle's her cursor around the point*] and have a conversation about if they're correct. And then, the nice thing is then having a conversation about how we can use this tracepoint and how all of the values on this line. We can help the graph emerge in students' thinking [*moves t-slider to illustrate the trace feature on the graph*] that's nice. I like that.

As Ebony continued interacting with the applet, she explained to me that she wanted a line segment on the graph to represent the change in the burnt length as she increased the t-slider. In response to her perceived constraint of the applet, Ebony constructed a new technique for conveying her intended image for the change in the burnt length by remembering the capabilities of Zoom and its annotation feature. She described that if she were to use this applet during instruction, she would press play on the applet and have students watch the emergence of the tracing of the graph in order to verify the numerical values on the constructed table that represents changes in the remaining length and burnt length relative to the changes in time. She used the annotation feature to draw horizontal vectors to represent changes in time and vertical vectors to represent the changes in the burnt length, which would result in changes in the remaining height of the candle.

Armando's initial interaction with the CB-Applet entailed trying to discover what the applet could do in relation to the task by typing in the original height of the candle that he produced and the CROC from Task 1 (e.g., 1/4 in per hr., but mistakenly typed 1.4). He began to demonstrate that if he used the applet during instruction, he would use the trace function to plot the points (6, 1.6) and (5,3) on the line and then determine the value of the CROC based on the definition. He suggested that if one would like to know the CROC between these two points (if the burn rate box was hidden), then one can compute the CROC by using the ratio ($\Delta r/\Delta t$ where r and h represent the remaining length of the candle). He explained that once students are able to calculate the CROC to be -1.4, then the mathematical expression $\Delta r/\Delta t$ can support their construction of a linear line through the idea of building a "staircase" (see Figure 5). His goal of building a staircase was to determine corresponding changes in r when the change in t is known as it can be used to produce the original height of the candle. Throughout his interactions with the applet, Armando appeared to be interested in the calculations needed to determine a CROC and its values to construct a graph, rather than thinking of how a student might understand their

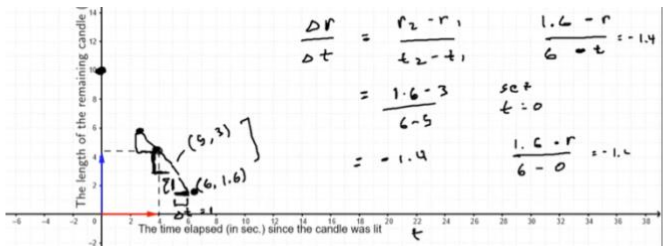

Figure 5: Armando's illustration of building a staircase

conveyed meanings for CROC. In addition, the questions he described that he would pose to his class when using the applet all focused students' attention on interpreting static information about the graph.

Discussions

Ebony's MMT with regard to Task 1 focused on engaging her students in conversations centered around thinking about the quantities, their relationship between the quantities, and how those quantities vary together. Then, when describing her use of the ShowTrace feature in the context of a classroom discussion, Ebony focused on a conceptual understanding of CROC as a way to describe the proportional relationship that exists between the changes in two quantities' values as they vary together. She used the dynamic visualization of tracing a point as representing a record of the relationship between covarying quantities (e.g., a graph that represents the time elapsed since the candle was lit and the remaining length of the candle as they vary together at a CROC of -¼ in. per hr.). Hence, her intended use of the applet as an instrument to support students understanding of CROC aligned with her expressed instructional goals for teaching CROC.

In contrast, Armando's MMT and orientation for Task 1 focused on expressing his meaning for CROC as a procedure for calculating other values that satisfy the relationship of the problem context. For him, key attributes of CROC were that the ratio $\Delta r/\Delta t$ expresses the slope of a line and indicates the graph's steepness. As he was describing his use of the ShowTrace feature in the context of teaching, Armando focused on the ratio $\Delta r/\Delta t$ as a way for computing coordinate points that can be connected by a straight (e.g., a way for illustrating the slope) and used the building of "staircases" to construct a linear graph. His perception of the applet was an instrument that can visually verify that the ratio for CROC as a secant line between two points on the graph of a function. Rather, Ebony perceived the intended use of the applet as an instrument that can support and sustain a conversation centered around student's conceptualization of the problem context.

The results from this study suggest that a teacher's conceptualization of how an applet's features can support students' mathematical learning is guided by their MMT. The mathematical meanings a teacher holds guide their instructional decisions and actions, which impacts how they use the applet during instruction. One limitation to this study is that only two instructors were presented. I intend to conduct a similar study on a larger scale, where further teacher research should focus on the teachers' conceptualization of the applet(s) as an instrument which impacts their implementation of the applet to support students' mathematical learning.

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