Building Knowledge within Classroom Mathematics Discussions

Jason K. Belnap
The University of Wisconsin—Oshkosh

Over the past decade, a growing trend has been to center instruction around student learning, rather than teacher performance. Such instruction often elicits student participation through various classroom activities: answering questions; solving problems; having students do board work; working on collaborative tasks in groups or pairs; making and testing conjectures; presenting ideas, proofs, and solutions; and debating. Through these and other classroom activities, students are expected to engage in the learning process, participate in mathematical thinking, and contribute to classroom discourse.

The presence of these instructional forms in the classroom does not always indicate quality instruction or guarantee quality student involvement. Cursory classroom observations might not reveal students' mathematical thinking or engagement. For example, working in groups, carrying on discussions, or answering questions does not necessarily mean that they are engaged in mathematical thought. We would need to know: What is the nature of the group's task? What are they discussing? How thought provoking are the questions? and What is the nature of students' responses?

In general, determining the quality of student involvement and their role in building classroom knowledge requires us to answer deeper questions about the discourse: How are learners contributing to the discussion? What is the nature of those contributions? What role are they playing in the discussion? and What significance and impact do their contributions have on the developing content? Addressing these questions for discussions in the mathematics classroom setting is the aim of this paper.

Within a different context and social group, I was recently able to address these exact questions, while developing an analytical framework. While studying discussions among practicing teachers who participated in a professional development program, Belnap and Withers developed an analytical framework for identifying the origin of a discussion's content and how each individual contributed to that knowledge (Belnap & Withers, 2010).

This framework is based on a view of discourse that integrates aspects of various learning perspectives: constructivism (Cobb, 1994; Cobb & Yackel, 1996; Ernest, 1996; Sfard, 1998; Zevenbergen, 1996), the social perspective (Cobb, 1994; Cobb & Yackel, 1996; Lerman, 1998, 2000; Sfard, 1998), socioconstructivism (Lerman, 1998, 2000; Cobb & Yackel, 1996; Cobb, Jaworski, & Presmeg, 1996), and agency (Walter & Gerson, 2007). This view is that discourse involves the mutual construction of both individual and social knowledge; it is a social activity shaped by participants' involvement. At the same time, participants willfully act and construct their own knowledge from their involvement in the discourse.

From this perspective, the discussion's text represents a form of social knowledge constructed by the willful actions of its contributors. Stemming from social linguistics and the work of Nassaji and Wells (2000), Wells (1996), the framework Belnap and Withers developed both grew from and illuminates this idea. It describes how each individual's contribution links to the contributions of other participants, building the conversation's content (Belnap & Withers, 2010; Belnap, 2010).

As detailed in Belnap (2010), when individuals take turns in a discussion, they make willful contributions to the growing text, making *moves*. In building the discussion's content, each move has a function, determined by its *action* (i.e. how it affects the growing text) and its *target* (i.e. any content receiving the action).

Based on function, there are 13 different move (or function) categories, clustered into five main groups or types: *anchoring, valuing, altering, requesting,* and *contentless* moves. *Anchoring moves* present new ideas, opening potential lines of discussion. *Valuing moves* address the value, validity, or correctness of existing contributions, focusing on assessing, supporting, refuting, or otherwise affecting the credibility of prior contributions. *Altering moves* develop the content of existing contributions by adding to, modifying, or clarifying it. *Requesting moves* (including, but not limited to questions) solicit content. Finally, *contentless moves* either do not directly develop a discussion's content or are counted as such.

This framework provides a means of ascertaining the discussion's content structure. Each move's action and target describes a linkage between moves. Using these linkages to chain moves together breaks the discussion into *fibers*, each representing the development of a single idea.

Using this framework allowed me to see both the structure and individual contributions' roles in content development. Identifying fibers allowed me to distinguish separate ideas or topics in complex conversations, facilitating the identification of productive conversations (i.e. those relevant to the purpose of the PD program). The distinctions among moves provided a means of identifying substantive contributions to the conversation; by counting this information for individual participants and contrasting the results, I ascertained the extent and nature of their involvement.

As a particular example, using this framework allowed me to determine information about participant involvement, individual conversational roles, and discussion characteristics in a recent analysis of one professional development session. Specific data and details are provided in Belnap (2010).

The framework provided an overview of participant involvement in the session. I found that all participants took an active role in developing the discussion's content. Each participant initiated some conversational fibers. All listened to and built off of the ideas of others. Finally, they each made efforts to explain and support their own and others' contributions.

On a more specific level, the framework revealed the nature and extent of individual involvement. I found that the facilitator's involvement mainly consisted of initiating and soliciting content; the extent of this involvement was limited, as he often took a back seat, avoiding direct control of the content, and allowing it to develop at the participants' discretion. Other participants took an active role in the conversation, with no one clear discussion leader. One (while not dominating the conversation) did play a leading role, initiating conversational fibers, conveying information, evaluating/refuting contributions, and integrating and building ideas, all more so than any other participant. By contrast, two participants seemed to hold back and contribute much less.

In addition to individual involvement, by revealing the conversation's composition, the framework provided the means of characterizing the discussion overall. The discussion could be characterized by conveyance, slight developing, and justifying a wide variety of ideas and opinions, with almost no discussion, change, and deliberation of ideas. A common pattern was that a conversational fiber consisted of an initial idea, justification with some addition of ideas or details, and then a topic shift. Little time was spent deeply investigating the many ideas initiated. Most content arose from spontaneous comments. There was a profound lack of inquiry and little disagreement and resolution of differences.

It is plausible that this framework can be used to answer similar questions and provide similar information regarding mathematical classroom discussions; this is the goal of this study. At the same time, differences in these contexts (the mathematics classroom verses a professional development program for teachers) are great, including: strongly rooted cultural norms, roles, responsibilities, and expectations; differences in the nature of the discussed content; and typical goals and objectives for the two contexts. With such strong differences, it is plausible that the framework may need to be modified to accurately reflect the content development of mathematical conversations.

Based on discussion and feedback from members of the research community, I am conducting a pilot study, to see how the framework can describe content development in a mathematics classroom. To do this, I have purposively selected a mathematics teacher who is well known for effectively engaging students in investigative tasks, orchestrating student centered classroom discussion, and establishing classroom discourse in which students listen to and respond to each other.

To test the analytical framework using typical qualitative methods. I will video tape an hour long class and apply the framework to the coding of the class' transcript, looking for contributions whose function may not be described well by the framework, modifying and reconceptualizing the move categories as necessary to account for these differences. Once I have completed analysis of the class, I will discuss the results with another researcher to gain an outside perspective and find concepts and ideas that may be missed, adjusting the framework as necessary. Next, to test the modified framework, I will apply the framework to the transcript of a second class to see if the framework accurately describes the discussion. Finally, I will analyze the resulting data to determine the extent to which the framework facilitates answering the questions posed earlier: How are learners contributing to the discussion? What is the nature of those contributions? What role are the students playing in the discussion? and What significance and impact do their contributions have on the developing content?

Data collection is currently beginning and preliminary results will be reached during December 2010 and January 2011. This paper and presentation will focus on discussing these preliminary results, examining potential information they give about the mathematics classroom, and beginning to contrast this with other frameworks that examine classroom discourse. As a preliminary presentation, participant discussion will also center on feedback, ideas, observations, and additional viewpoints on these same three issues. In particular, I will pose questions for discussion such as: What useful information can this framework provide for us as researchers? What interesting research questions could be answered utilizing this framework? How may this framework relate to other analytical or theoretical work? and What details may I have overlooked?

References

Belnap, J. K. (2010). Building classroom mathematical knowledge: Extending a framework for the co-construction of knowledge to the classroom context. In *Proceedings of the thirteenth conference for Research in Undergraduate Mathematics Education.* Raleigh, NC.

Belnap, J. K., & Withers, M. G. (2010). *Discourse analysis: Finding structure in the content of uncontrolled discussions.* Manuscript in preparation, University of Wisconsin at Oshkosh.

Cobb, P. (1994). Where is the mind? constructivist and sociocultural perspectives on mathematical development. *Educational Researcher*, *23*(7), 13–20.

Cobb, P., Jaworski, B., & Presmeg, N. (1996). Emergent and sociocultural views of mathematical activity. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 3–19). Mahwah, NJ: Lawrence Erlbaum Associates.

Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist*, *31*(3/4), 175–190.

Ernest, P. (1996). Varieties of constructivism: A framework for comparison. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 335–350). Mahwah, NJ: Lawrence Erlbaum Associates.

Lerman, S. (1998). Research on socio-cultural perspectives of mathematics teaching and learning. In A. Sierpinska & J. Kilpatrick (Eds.), *Mathematics education as a research domain: A search for identity* (p. 333-350). Great Britain: Kluwer Academic Publishers.

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 19–44). Westport, CT: Ablex Publishing.

Nassaji, H., & Wells, G. (2000). What's the use of 'triadic dialogue'?: An investigation of teacher-student interaction. *Applied Linguistics*, *21*(3), 376–406.

Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational Researcher*, *27*(2), 4–13.

Walter, J. G., & Gerson, H. (2007). Teachers' personal agency: Making sense of slope through additive structures. *Educational Studies in Mathematics*, *65*(2), 203–233.

Wells, G. (1996). Using the tool-kit of discourse in the activity of learning and teaching. *Mind, Culture, and Activity*, *21*(2), 74–101.

Zevenbergen, R. (1996). Constructivism as a liberal bourgeois discourse. *Educational Studies in Mathematics*, *31*, 95–113.

Using Think Alouds to Remove Bottlenecks in Mathematics

Kavita Bhatia[1] and Kirthi Premadasa[2]

[1]University of Wisconsin Marshfield/Wood County, Marshfield, WI and

[2]University of Wisconsin Marathon County, Wausau, WI

Abstract

Think alouds are a research tool originally developed by cognitive psychologists for the purpose of studying how people solve problems. The basic idea being that if a subject can be trained to think out aloud while completing a certain task then the introspections can be analyzed and may provide insights into misunderstandings as well as higher thinking. This talk is a preliminary report of a think aloud conducted with calculus students to understand their difficulties with work problems in integral calculus.

Keywords: Calculus, cognitive science, classroom research, think alouds

Students in integral calculus often face difficulties in problems involving applications to physics like work and pressure problems. It is unclear whether their difficulties are due to lack of understanding of the definition of the definite integral as a limit of Riemann sums or whether it is difficulty in actually applying the concept to a physical situation like a work problem. This study is a preliminary report of an investigation conducted using the think aloud (or protocol analysis) technique with second semester calculus students at a two year campus of the University of Wisconsin.

Think alouds are a research tool originally developed by cognitive psychologists for the purpose of studying how people solve problems. The basic idea being that if a subject can be trained to think out aloud while completing a certain task then the introspections can be analyzed and may provide insights into misunderstandings as well as higher thinking. Schoenfeld(1) has used verbal transcripts and protocol analysis to study mathematical problem solving .

The goal of this study was to answer the question “Why do calculus II students have difficulty solving Work problems?”. This was a qualitative study. Six students of varying abilities were selected to participate in the study. Students were first trained to think aloud by being asked to solve simple linear equations. They were then given a series of work problems ranging from the simplest kind with a fixed force and a fixed distance to the more involved that had a variable force and/or a variable distance. The sessions were video recorded. During the sessions the students were not prompted in any way and nor were any interventions introduced. The only comments made by the instructor were to request the student to verbalize their thoughts if and when the student fell silent. The recordings were transcribed and screen shots of the diagrams were taken. The transcriptions were coded and analyzed.

A coding scheme (see Appendix 1) was developed to code the verbalizations using Polya's four step problem solving process. Next we needed to identify the codes that would enable us to answer our question. We highlighted three codes, strategy, mathematical argument and logical inference. These codes reflect the thought process of a mathematician while problem solving. Each problem was then assigned a rating from 1-5 for each of the selected codes using a rubric (see Appendix 2 for the rubric), with 5 representing the score of a mathematician and 1that of a novice problem solver. These ratings helped to identify possible bottlenecks in the problem solving process.

Preliminary analysis indicates that the students who got "1" under strategy had a common trait. They headed straight for the fluid slice in the pool type problems but were then confused as to what to do next. Therefore one possible bottleneck is students memorizing a fragment of the instructor's strategy without understanding the underlying connections. Four of the six students seemed to have a significant strategy but were unable to solve the problem correctly due to mistakes in calculating the volume of a generic slice or incorrectly calculating the weight of the slice. This suggests that students do not have a good handle on the basic mathematical tools that are considered essential at this level.

During regular assessment students often erase their wrong work, so we only see the end product which doesn't always help us to identify the bottleneck. With a think aloud we are able to see much more of the problem solving process, the students' struggles in formulating strategies and mathematical arguments and thus make the thinking process more visible.

Questions for discussion

1. Can we make our think aloud coding list portable for problem solving across the university mathematics curriculum?
2. As an intervention for lack of strategy, will a grading rubric which will ask students to actually write down the strategy (before starting the mathematics), encourage students to strategize more ?
3. Strategy, Mathematical Argument and Logical Inference are key thought processes of a Mathematician solving problems. Are there any others that should be considered in this analysis?

References

1) Lendol Calder and Sarah-Eva Carlson, "Using Think Alouds to Evaluate Deep Understanding", Handout at Faculty College 2009, Office of Professional and Instructional Development, University of Wisconsin System.
2) Alan Schoenfeld, "Mathematical Problem Solving", Academic Press, Inc, 1985.

Appendix 1: Coding Sheet based on Polya's four steps

A. Understanding the problem

I. Understanding the problem UP
II. Recall RE

B. Devising a plan

I. Initial Plan DP
II. Alternate Plan DAP

C. **Carrying out the plan**

I. Setting up the variable SV
II. Mathematical argument MA
III. Mathematical argument (In correct,) MA_I
IV. Questioning (Q)
V. Guess (G)
VI. Recognizing limits (H)
VII. Narration (N)
VIII. Uncategorized (X)
IX. Strategizing (ST)
X. Strategizing with Reflection (ST_R)
XI. Inference (geometry) I_G
XII. Inference (Reflexive) I_R
XIII. Inference (Previous) I_P
XIV. Rearranging Terms R_T
XV. Calculation (C)

D. Looking Back

XVI. Revision (R)
XVII. Reflection on concept (R_C)
XVIII. Reflection(Rf)

Appendix 2: Mathematical Argument Rubric

Representative Thought	Points
Focal mathematical Argument executed(correct) early, i.e. Mathematical procedures applied correctly at the appropriate steps to solve the problem correctly	5
Focal mathematical Argument executed(correct) , late Or Focal Mathematical Argument executed (correct)early except for non-conceptual mistakes	4
Mathematical argument with some focus correctness but has significant mistakes Could not completely carry out mathematical procedures	3
Mathematical argument , with little focus with minor parts which are correct	2
Unfocussed Mathematical argument containing a few correct components.	1

Strategy Rubric

Representative Thought	Points
Focal Strategy achieved early (professional)	5
Focal Strategy achieved	4
Focal Strategy achieved with uncertainties	3
Indication of a strategy in the problem solving process but is not the focal strategy nor does it contain parts of the focal strategy	2
Unfocussed /unsignificant strategy , i.e.No evidence of a strategy or procedure	1