STUDENT UNDERSTANDING OF INTEGRATION IN THE CONTEXT AND NOTATION OF THERMODYNAMICS: CONCEPTS, REPRESENTATIONS, AND TRANSFER

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Students are expected to apply the mathematics learned in their mathematics courses to concepts and problems in physics. Little empirical research has investigated how readily students are able to transfer their mathematical knowledge and skills from their mathematics classes to other courses. In physics education research, few studies have distinguished between difficulties students have with physics concepts and those with either the mathematics concepts, application of those concepts, or the representations used to connect the math and the physics. We report on empirical studies of student conceptual difficulties with (single-variable) integration on mathematics questions that are analogous to canonical questions in thermodynamics.

Key words: [Physics, integrals, conceptual understanding, representations, transfer]

Introduction

Mathematics is a vital part of how physics concepts are represented (e.g. equations, graphs and diagrams) and how problems are solved, all across the curriculum. It allows students to simplify the analysis of complex problems by representing complicated conceptual physics problems as a relatively simple relationship between variables. Appropriate interpretation of these representations requires recognition of the connections between the physics and the mathematics built into the representation as well as subsequent application of the related mathematical concepts (Redish, 2005).

Students are expected to apply the mathematics learned in their mathematics courses to concepts and problems in physics. Despite the fact that students are expected to carry out such interdisciplinary study as a matter of course, little empirical research has investigated how readily students are able to transfer their mathematical knowledge and skills from their mathematics classes to other courses.

With many physics topics, specific mathematical concepts are required for a complete understanding and appreciation of the physics. Meltzer (2002) has shown a link between mathematical acumen and success in an algebra-based physics class. Tuminaro and Redish (2007) have combined the frameworks of resources (Hammer, 1996), epistemic games, and framing to analyze student use of mathematics in physics. To date, however, there have only been a few physics education research (PER) studies exploring physics students' difficulties with calculus concepts (Black and Wittmann, 2009; Cui, Rebello, and

Bennett, 2007; Pollock, Thompson, and Mountcastle, 2007; Rebello, Cui, Bennett, Zollman, and Ozimek,
2007).

Our work aims to identify the extent to which mathematical understanding affects physics conceptual knowledge, specifically in the context of upper-level thermal physics. We have two main research questions: What specific difficulties do advanced-level undergraduate students have when learning thermal physics concepts? To what extent do students' mathematical understandings influence their responses to physics questions?

Background

One mathematical concept that has emerged as an area for investigation in our research is that of integration. In thermodynamics, two- or three-dimensional graphical representations of physical processes are especially useful in helping to understand these processes. These diagrams can contain information about the thermodynamic "path" followed in a process, regions of different phase, and critical behavior. Diagrams that plot pressure *versus* volume, known as *P-V* diagrams, are canonical representations of physical processes as well as of the corresponding mathematical models in thermodynamics. Information can quickly and easily be obtained from these representations. A point on this diagram represents the pressure and volume values for a particular equilibrium state of a system. Thermodynamic processes (reversible processes, at any rate) can be represented as curves on the diagram tracing the "path" the system took to go from its initial state to its final state during a specific process. One important physical quantity that can be obtained from this diagram is the thermodynamic work done (energy transfer via mechanical forces) on a system undergoing a thermodynamic process. The work done on the system is defined as the integral of pressure with respect to the volume:

$$\mathrm{W} \equiv \int P dV$$

Thus physicists can quickly determine the work done by a calculation, if the function *P(V)* is known for a given process; qualitative determinations can be made by evaluating the area under the curve for a particular process.

Questions that have focused on student understanding of physics and that have featured integration have proved troublesome for students at the introductory level (Meltzer, 2004). Meltzer gave students a *P-V* diagram in which an ideal gas undergoes two different thermodynamic processes, but each process starts in the same state and ends in the same state. For the situation described, he figure is shown in Fig. 1(a). The most common incorrect response and reasoning was that the works were equal, based on the initial and final states being the same for each process. There is a reasonable physics-based interpretation for this reasoning, namely that students are effectively treating work as a function of state (which would only depend on the "endpoints") rather than a process-dependent quantity.

Previous findings on student understanding of integral calculus concepts in research in undergraduate mathematics education indicate that students do not possess the necessary knowledge to allow them to successfully complete problems involving concepts of integration, especially with regards to considering the integral as the area under the curve. The literature in mathematics education repeatedly documents the lack of student understanding of the relationship between a definite integral and the area under the curve (Grundmeier, Hansen, and Sousa, 2006; Orton, 1983; Thompson, 1994; Vinner, 1989). These include student difficulties with recognition of integrals as limits of

(Riemann) sums (Orton, 1983; Sealey, 2006); student confusion about the concept of "negative area" for integrals of curves that fall below the *x*-axis either conceptually (Bezuidenhout and Olivier, 2000), computationally (Orton, 1983; Rasslan & Tall, 2002) or both (Hall, 2010). Thompson and Silverman (2007) showed that the reliance on *area under curve* reasoning may limit applicability of the conception of integrals.

Researchers at the University of Maine asked Meltzer's questions in upper-division thermal physics courses and found similar difficulties and reasoning patterns that Meltzer found (Pollock, Thompson, and Mountcastle, 2007). We wondered if some of the conceptual difficulties might originate in the confusion over the mathematics involved. In addition to replicating Meltzer's experiment in our upper-level thermodynamics course, we designed qualitative questions regarding comparisons and determinations of the magnitudes and signs of integrals, without any physics context, that are analogous to Meltzer's questions. We call these "physicsless physics questions" (Christensen and Thompson, 2010). The physicsless physics version uses notation that is more consistent with representations used in physics than math. By asking the physicsless phyiscs question we hope to further isolate mathematical difficulties that may underlie observed physics difficulties.

In the physicsless physics question related to Meltzer's work question, students are asked to determine the sign of the integral of each of two functions from the same starting point (*a*) to the same ending point (*b*), and to compare the magnitudes of the integrals:

$$I_1 \equiv \int_a^b f(y)dy \,,$$

$$I_2 \equiv \int_a^b g(y)dy \,.$$

One version of the figure provided to students for this question is shown in Fig. 1(b). We administered the analogous math version of the question as well as the physics-less questions to the students in our upper-division thermodynamics courses (Christensen and Thompson, 2010). Many of the students used "endpoint" reasoning to state that the integrals were equal, in complete analogy to the reasoning seen in physics questions.

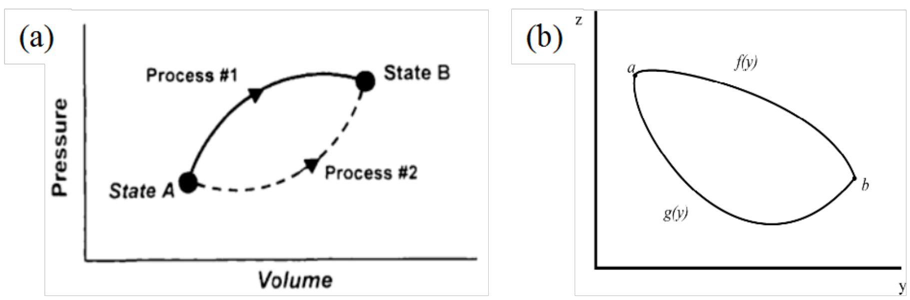

Figure 1. Diagrams used on questions described in the text. (a) Pressure-Volume diagram used to compare work done by identical systems over two different thermodynamic processes, from Meltzer (2004); (b) one version of figure used in "physicsless physics question" to compare values of the integral of the two functions shown over the same interval.

The survey results from the paired physics and physicsless questions among physics students show that some of the difficulties that arise when comparing thermodynamic work based on a pressure-volume (*P-V*) diagram may be attributed to difficulties with the *mathematical* aspect of the diagram, in particular with the correct application of an understanding of integrals, rather than (or in addition to) physics conceptual difficulties.

The original target population for our earlier research was student in upper-division physics courses. However, as we explored the mathematical aspects in more detail, we have included students in mathematics courses as part of our study as well. We also asked the physicsless physics question to students at the end of the third semester of calculus: a greater proportion of calculus students used "endpoint" reasoning than did physics students.

The current study extends earlier work in two ways. First, we follow up on the results from the physicsless physics question in calculus classes. We explore the extent to which explore students' understanding of the mathematics using a more conventional graph to start with. By removing the unfamiliar representation, we can start to determine the extent of students' ability to interpret graphs in the context of integration.

Experiment Design

We use the empirical framework of *specific difficulties* (Heron, 2003) to guide analysis of data and descriptions of student reasoning in our research. We start with targeted, context-dependent results and then generalize across contexts, seeking larger patterns of student responses in our data. Our emphasis is on gathering and interpreting empirical data that can act as a foundation for future studies on reasoning in physics and for curriculum development to address specific difficulties.

The original target population for our earlier research was student in upper-division physics courses. However, as we explored the mathematical aspects in more detail, we have included students in mathematics courses as part of our study as well. The study described here includes students in single-variable integral calculus (Calculus 2) and in multivariate calculus (Calculus 3). We were also curious about the extent to which introductory math students may have a different concept image of integrals, or certain facets of integrals, than physics students who recently learned the requisite math concepts; thus we also included students in our study who were taking the second semester of the calculus-based introductory physics course (physics 2), who had taken previously or were taking concurrently calculus 2.

It was suggested by our colleagues in the RUME community in particular that features in the physicsless physics question (Fig. 1(b)), because they lack the conventions from mathematics for a representation of an integral of a single-variable function, may be cueing students to try different mathematical approaches, in particular analysis of the task using line integrals. Following up on these suggestions and others from previous work, we created an analogous math version of the question (Fig. 2) that would provide a more mathematically acceptable representation for the question. In particular, the analogous math version of the question uses y and x as the axis labels, has each function extend beyond the intersection points, and shows the limits of the integral on the x-axis, as values of x rather than points on the graph.

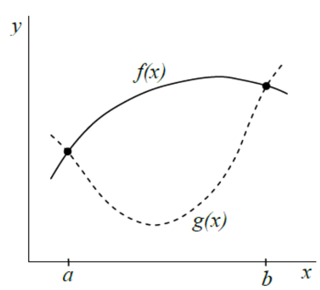

Figure 2. Diagram used for the Analogous Math version of the question described in the text.

We report here on two separate but related studies as extensions of the work reported earlier (Christensen and Thompson, 2010). In the first study, we gave the physicsless physics question to students in the last week of instruction in both Calculus 2 and Calculus 3, to see if the representation cued different knowledge in the different courses. To follow up on previous results and to see if different disciplinary populations would produce different results, the analogous math question was given near the end of the semester to a separate group of Calculus 3 students as well as a group of students at the end of Physics 2. Responses to the physicsless physics question and the analogous math question were analyzed for patterns and categorized.

To probe more deeply into the stability and the nature of students' concept image of these integral questions, seven interviews were carried out, using a think aloud protocol (Ericsson and Simon, 1998). These interviews were designed to build on the replies from the survey results. Interview subjects were taking physics 2; interviews occurred near the end of the semester. Each of the 7 subjects was asked to solve two of the written survey questions at the start of the interview. The interviewer then questioned students in order to follow up on their responses, and asked them to explain their reasoning in more detail. Based on the category of reasoning students provided in the initial conversation, the interviewer asked specific follow-up questions taken from a large set of possible questions. The follow-up questions were chosen to probe the stability and the nature of the reasoning that students were using to answer the first question.

Written Survey Data

The physicsless physics question shown in Fig. 1(b) was given to 41 students in calculus 2 and 56 students in calculus 3. When asked about the sign of the integral, just over three quarters of the calc 2 students and just over two thirds of the calc 3 students were correctly able to identify both integrals as being positive (first two rows of Table 1). When asked to compare the magnitude of the integrals for *f(y)* and *g(y)*, about 90% of calc 2 students were able to answer the question correctly, while about 60% of calc 3 students were able to correctly answer the question (first two columns of Table 2).

Table 1. Results for students who answered that both integrals are positive for the different versions of the integral question.

	Both Integrals Positive
Physicsless Calc 2 (N=41)	78%
Physicsless Calc 3 (N=56)	68%
Analogous Math Calc 3 (N=95)	77%
Analogous Math Physics 2 (N=95)	78%

Table 2. Results for magnitude comparison of the integrals in the written surveys.

	Physicsless Physics Calc 2 N=41	Physicsless Physics Calc 3 N= 56	Analogous Math Calc 3 N=95	Analogous Math Physics 2 N=95
Greater	**87%**	**58%**	**73%**	**73%**
Less than	0	2	7	3

Equal	3	7	9	1
Not enough	0	9	9	3
Other/No Ans	1	8	1	7

One interesting finding is that calculus 2 students markedly outperformed calculus 3 students in the magnitude comparison task. One explanation of the difference comes from the reasoning students gave. About 20% of calc 3 students said that I_1 was less than I_2, while none of the calc 2 students gave this response. A significant fraction of the reasoning given for this response in calc 3 was that *g(y)* appears to be longer in length than *f(y).* This is consistent with the idea that some students are interpreting the task as the evaluation of a line integral, an idea further supported by the absence of such reasoning in the calc 2 population, who have not yet learned about line integrals. Additional reasoning for *less than* includes a comparison of the "curvatures" (i.e., second derivatives) of the two curves: since *g(y)* has a larger magnitude of curvature, it has a greater integral. We note that a far smaller fraction of students said that the integrals would be equal. We attribute this to the asymmetry of this version of the question, since previous versions with symmetric curves (as in the physics version) yielded higher fractions of *equal* responses. Overall these results suggest that the details of the representation affect the responses. This has strong implications for the teaching of the representations in physics classes.

In a separate survey, the analogous math question was given to 95 students in calculus 3 and a separate 95 students in physics 2. We find that in both classes, over three quarters of the students are able to correctly determine the signs of both integrals, and about three quarters of the students are able to correctly compare the magnitudes of the integrals. The reasoning given with answers was not always clear, but they could roughly be put into a number of categories: area under the curve, position of the function, and shape of the curve (which includes both the slope and the curvature). Just over 50% explicitly used the word "area" in their reasoning. As mentioned above, while there is evidence for these categories of reasoning, the written data do not provide definitive proof that students have a concept image that is consistent with their explanations. The interviews provided more insight into student reasoning categories.

Interview Results

We found it difficult to understand the concept image (Vinner, 1989) that students from the survey answers alone. While 50% of students use the term *area* to describe the area under the curve, in many other answers it is hard to tell whether area under the curve is a part of the concept image. To probe more deeply into the stability and the nature of students' concept image of integrals in these questions, seven interviews were carried out, each with a think-aloud protocol (Ericsson and Simon, 1998). These interviews were designed to build on the responses from the survey results.

We are currently analyzing the seven interviews. Early into observing the interviews two observations have been notable: (1) students are unable to use mathematical reasoning to explain why the integral in the negative-*y* direction is negative when looking at the diagram, (2) while unable to explain the negative area under the curve mathematically, students are more comfortable talking about negative area – the area when the function is below the *x*-axis – when discussing the question in a physics context.

Interpretation of sign of integral by reversing the direction of integration

Freddie, one of the participants in the interviews, correctly identifies that the integrals are both positive. The interviewer then asks Freddie what the sign of the integral is if the integration direction is reversed (i.e., so that the integral is taken from *b* to *a* instead of *a* to *b*). Given a positive function, this method will yield a negative integral.

I: "So if I ask you to compare the integration from this point [points to f(b)] to this point [points to f(a)]." [The interviewer then writes $I \equiv \int_b^a f(x)\,dx$ *on the board.]*

F: "Ok so... Given that the integral, the way you have it set up from what I can tell, is used to find the area under the curve it should still be positive…"

Using the graphical representation, Freddie is fairly certain that the reversed-direction integral is still positive. But then Freddie incorrectly invokes the Fundamental Theorem of Calculus, comparing the endpoint values of the function (*f(x)*, the integrand) rather than the integrated function (*F(x)*). For the function Freddie was given, *f(a)* was less than *f(b)*, so that the difference would be negative. Freddie uses this to state his response:

F: "Based on that it would look like it would probably be negative."

Another student, Simon, also correctly identifies the initial integrals as positive. When asked to consider the reversed-direction integral, Simon explicitly uses the Riemann sum to think about the sign of the integral. He draws "columns" and states that the width of the columns are "your little *dx*s," and recognizes the arbitrariness of *dx*: "you can change your dx depending on, you know, whether or not you want to change by so much or so little…"

Further discussion of the process of integration elicits some (also incorrect) ideas about the Fundamental Theorem of Calculus (FTC) and the Riemann sum concept, in which he states that if "your last [column] is then […] smaller than your first [column], then I would, I would think you are going to get a negative value [for an integral]." Like Freddie, Simon confused *f(x)* with *F(x)* in invoking the FTC and came up with the correct sign for an incorrect reason. But again, Simon is conflicted between the answers obtained using graphical and symbolic representations.

The interviewer prods Simon more about the situation to get a sign for the reversed-direction integral. Simon responds:

S: "I feel that it should be positive because technically it shouldn't matter how you count this together right? … If you counted this way [moving his hand from right to left] or you count this way [moving his hand from left to right across the diagram] and you keep the dx *the same, you should find the same area."*

Simon seems to be adding the "columns" used for the Riemann sum in his description. For Simon, these columns seem to represent positive quantities, independent of the direction.

Both Freddie and Simon use the FTC to reason that the reversed-direction integral is negative, and also use the graphical represetation to reason that the area under the curve is positive regardless of integration direction.

Interpretation of sign of integral below the x-axis using math and physics contexts

Freddie was asked about the sign of the integral for a function whose values were negative over the range of integration (but the direction of integration was in increasing *x*). He invoked *area under curve* reasoning to interpret the integral on the graph, but initially he had difficulty in identifying the integral as negative in this case. When presented with a sine curve, Freddie stated the following while contemplating the negative part of the integral:

F: *"In order to get negative area it is not... conceptually, looking at like a plot of land, it would be an impossibility. However, we are looking at something like a voltage; voltages can very easily go negative because we only have them in reference to what we called to be ground."*

In this case Freddy felt more comfortable using voltage to interpret the negative area under the curve than giving a mathematical explanation. Freddy also used the equation to infer that the area under the curve must be negative, while initially looking at the diagram: he felt that the area must be negative. The reasons given by students in interviews show that graphically, students find it difficult to account for a negative integral using only mathematical knowledge, but when physical meaning is attributed to the variables in the graph, students are better able to interpret the integral of a negative function.

Conclusions

The survey results build on the work of Christensen and Thompson (2010) to show that students entering thermodynamics have non-trivial difficulties with the math they need to understand systems that are represented like the one in Figure 1(a). One of the reasons for the revised surveys was to test how the students' knowledge of calculus might affect their responses to the physicsless physics and analogous math questions. Calculus 3 students were outperformed by Calculus 2 students, most likely due to the additional knowledge of line integrals brought to bear on the task by some of the calc 3 students. Our preliminary results suggest that calculus 3 students performed better on the analogous math question than on the physicsless physics question. This is consistent with the idea that the features of the analogous math question leave less ambiguity about the nature of the representation itself, and thus students are less likely to misinterpret the task as a line integral task. However, the sample sizes are not large enough to make more than tentative conclusions. Further surveys will be asked this year to follow up on the ones already asked.

We have made a preliminary review of the interviews, with specific emphasis on aspects of student interpretations of negative integrals, either as integrals of positive functions taken from a higher-valued limit to a lower-valued one (i.e., a "right-to-left" or reversed-direction integral) or as integrals of functions that are negative within the range of integration (i.e., integrals of negative functions).

Prevalent throughout the interviews was attention to the idea of reversing the direction of the integral, which is a relevant procedure in some physics contexts. Prior research has not addressed student interpretation of this method, however. Interview results suggest that most students struggled with this task. One possible reason for this may be students failing to think of the differential *dx* as a negative quantity when the direction of the integration is reversed. This certainly appeared to be the case more explicitly for Simon's example. There are implications here for student understanding of the Riemann sum and the integral as the limit of that sum. Additionally, we find students incorrectly invoking the Fundamental Theorem of Calculus to reason about the sign of reversed-direction integrals. (This particular confusion with applying the FTC to values of the integrand has been seen by Pollock, and is reported in his thesis (Pollock 2008).)

The reasons given by students in interviews show that graphically, students find it difficult to account for an integral of a function that is negative within the range of integration using only mathematical knowledge; this result is consistent with literature from the RUME community (Bezuidenhout and Olivier, 2000; Hall, 2010; Orton, 1983;

Rasslan & Tall,

2002). However, students who attribute physical meaning to the variables in the graph are better able to interpret this type of integral and make sense of the negative "area."

We will pursue these observations in more depth with newly revised survey questions and with further analysis of these interviews. In this analysis, we will also follow up on how students use physics concepts compared to math knowledge when discussing the idea of negative area, as seen in Freddie's interview.

Finally, we feel that this work has relevance to the cognitive framework of *transfer in pieces* (Wagner, 2006), ideally suited for the study of knowledge transfer and the context-sensitivity of mathematical knowledge. The interviews to date have largely focused on students' mathematical ideas, and the analysis has focused on student knowledge of content and representations. We plan to extend the analysis and to carry out additional interviews, to probe students' transfer between mathematics and physics, through the transfer-in-pieces lens.

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