# THE INTERNAL DISCIPLINARIAN: WHO IS IN CONTROL?

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*A group of mathematicians and mathematics educators are collaborating in the fine-grained examination of selected 'slices' of video recordings of lectures drawing on Schoenfeld's ROG framework of teaching-in-context. We seek to examine ways in which this model can be extended to examine university lecturing. In the process we have identified a number of lecturer behaviours. There are times when, in what appears to be an internal dialogue, lecturing decisions are driven by the mathematician within the lecturer despite the pre-stated intentions of the lecturer to be a teacher. We analyse three scenarios: in the first the teacher prevails, in the second the mathematician, and in the third the mathematician appears to prompt the teacher to be more rigorous. We analyse these behaviours against the lecturers' ROGs, both stated and inferred. The value of the approach used as a professional development model is considered.*

## Introduction

Delivering mathematics content to large numbers of students via the medium of lectures presents a number of pedagogical difficulties that are seldom explicitly addressed (Speer, Smith, & Horvath, 2010). This paper presents a small part of a two-year project examining the feasibility of a professional development model, with two key aims. The first is to investigate whether Schoenfeld's Resources, Orientations and Goals (ROG) theoretical framework (see details below) can be adapted to analyse university mathematics lecturing. The second broad question is whether an effective lecturing professional development strategy can be built around a community of practice focussed on an examination of lecture practice. Our primary hypothesis related to these aims is that a personal awareness of our resources, orientations and goals, and a willingness to make changes to them, are major catalysts for professional development (PD) as a lecturer (and indeed as any teacher).

## Theoretical Framework

Schoenfeld (1998, 2002, 2007, 2008, 2011) and his Teacher Model Group (TMG) at the University of California at Berkeley have developed a theoretical framework of teaching-in-context, with a goal of answering how and why teachers make the in-the-moment choices they do while they are engaged in the act of teaching. The current version of the framework (Schoenfeld, 2011) is based on Resources, Orientations and Goals (ROG) that teachers bring to their practice. One formulation of the process is that "what a teacher decides to do while engaged in teaching is a function of the teacher's goals (some of which are determined prior to the instruction and some of which emerge as the lesson unfolds), beliefs (which serve to re-prioritize goals as some goals are satisfied or new goals emerge), and knowledge (including various routines the teacher has for achieving various goals)" (2008, p. 9). In the current discussion of the framework beliefs have been replaced by orientations, including dispositions, beliefs, values, tastes and preferences, and knowledge by resources (although still with an emphasis on knowledge). Thus when a teacher enters a classroom they use their orientations to adjust to the situation. Then goals are established based on the orientation, and relevant knowledge (R) is activated. Decisions consistent with the goals are made, consciously or unconsciously, about the directions to pursue and the resources to use (Schoenfeld, 2011). In an earlier paper he argues that:

> Teaching, …… depends on a large skill and knowledge base … its practice involves a significant amount of routine activity punctuated by occasional and at times unplanned but critically important decision making – decision making that can determine the success or failure of the effort. (Schoenfeld, 2007, p. 33)

These decisions made in the classroom are crucial. Since they are often made in-the-moment, yet "The quality of people's decision making…affects how successfully people attain the goals they set for themselves." (Schoenfeld, 2011, p. 36), analysing these decisions should be a vital part of a professional development programme. However, rather than being a single coherent set, an individual's ROG may likely, in any teaching situation, contain competing goals each inspired by a teacher's various orientations. It is this latter situation that is the subject of this paper, as we sought to analyse how the conflict of competing goals arising from an internal dialogue between mathematician and teacher were resolved.

## Professional Development of Mathematics Lecturers

While the effectiveness of various approaches to professional development in the K-12 domain has been extensively examined, comparatively little is known at the collegiate level (Speer et al., 2010). Addressing this need, in this project we structured community interactions to prioritise three practices identified as effective. Firstly we focused on "small, but meaningful, aspects of practice" (Speer, 2008, p. 219), "at the very level of detail when development and change appear to occur—the moment-to-moment decisions and practices of teachers." (Speer, 2008, p. 263). Such a fine-detailed examination, called microteaching, has been successfully used in teacher education and has been shown to influence student teacher achievement, with an effect size of 0.88, which is remarkably high compared to the "typical" effect-size of 0.40 (Hattie, 2009). Secondly, all discussions within the group followed the protocol that these should develop from concerns identified by the lecturers themselves. This aligns well with Robinson (1989), who argues for an empowerment paradigm in professional development that recognises the teacher as a professional and works from the 'bottom-up', providing teachers with opportunities to make meaningful choices, and Barton and Paterson (2009), who showed that teachers evidenced positive changes when working on self-identified areas of concern in their practice. Finally the development of a community of practice was actively fostered (Buckley & du Toit, 2010). In this forum common tools and language enable the development of shared meanings and sharing of tacit knowledge. In a manner similar to Jaworski, Treffert, and Bartsch (2009) we sought to "establish a collaborative process in which insider and outsider researchers both study the practices … in mathematics teaching-learning and use the research as a basis for understanding and reconsidering the practices involved" (Jaworski et al., p. 250), and argue that classroom-based enquiry can contribute to the "development of an inquiry community where research into teaching becomes a regular part of teaching practice and the community becomes more knowledgeable about its teaching" (Jaworski et al., p. 254).

This paper reports on an aspect of a project that explores how Schoenfeld's ROG framework may be used to direct lecturers' attention to aspects of their decision-making in the lecture theatre as a professional development activity. The project is informed by research concerning how a teacher's knowledge, orientations, or beliefs, and goals impact on their teaching practice (Schoenfeld, 2007; Ball, Bass & Hill, 2004; Shulman, 1986; Speer, Smith & Horvath, 2010; Törner, Tolka, Rosken & Sriraman, 2010). However, these are studies of teacher practice in primary and secondary schools and similar work at the college level is 'virtually non-existent' according to Speer et al. (2010). The project is also designed to build on the effectiveness of communities of practice (Lave & Wenger, 1991) and a culture of enquiring conversation (Rowland, 2000) for professional development. The project is part of a larger research study, which is described in more detail in Thomas, Kensington-Miller, Bartholomew, Barton, Paterson, and Yoon (under review).

The project aims to examine ways in which Schoenfeld's model can be used, and extended to examine university lecturing and to support the professional development of lecturers (Van Ort, Woodtli, & Hazard, 1991). In the process we have identified a number of lecturer behaviours, one of which is discussed in this paper. We have observed a number of instances of what appears to be an inner argument, or regulation by an inner voice, in the lecturer's communication with the class. In subsequent group discussion it has become clear that many lecturers are aware of this. In this paper we present three ways in which this tension plays out in the decision making process.

## Method

In this study a group of four mathematicians and four mathematics educators, all from Auckland University, have formed a community of practice to re-examine lecturing practice by collaborating in the fine-grained examination and discussion of lecturer actions in video recordings of lectures (Kazemi, Franke, & Lampert, 2009; Prushiek, McCarty, & Mcintyre, 2001). Lectures were video-recorded and the lecturer then chose a small section of less than five minutes that the whole group watched and discussed together in a fully supportive manner. This focussed discussion was audio-recorded and later transcribed, along with the brief sections of the lecture. We found that the discussion often started with the video content but then moved on to examine other relevant, related issues of learning, practice and mathematics. The video and audio transcription data was supplemented by an observer record, and a written lecturer-ROG (a statement by the lecturer of the knowledge used, orientation held, and goals, both specific goals intended for the lecture and more general educational goals) and interviews. The data was analysed by focussing on the relationship between decision points and the ROGs.

## Descriptions of the Three Scenarios

The three situations we describe involved three of the mathematicians, who were experienced lecturers, two male who we call Sandy and Simon and a female called Abi. Sandy presented an applied mathematics lecture to first year students, Simon a post-graduate lecture in number theory, and Abi a general education first year lecture on the Fibonacci sequence.

### *Sandy – The teacher takes control*

The primary purpose of Sandy's lecture was to consider solutions, for various values of the parameter $q$, of a difference equation that reduced to the form $x_m = qx_{m-1}\left(1-x_{m-1}\right)$. Sandy revealed a number of orientations that are relevant to this lecture. Firstly, as a teacher he values demonstrating results, and doing so without prior knowledge of precisely what may occur:

O1: "It is good to demonstrate things to the students rather than just tell them."

O2: "I'm pretty happy to experiment in this course and things will occasionally go wrong when you do that."

While he is doing so he wants the students to follow closely, without distractions:



O3: "I hope that my slides, being based on the course-book, allow students to follow what is going on without the distraction of extensive note taking"

He also has a pedagogical belief that, since he is part of a teaching team:

O4: It is important to stick to the course book and cover all the material.

With regard to the use of Matlab, his orientations were:

O5: It has value for exploration "I just wanted to explore, show them the graphs on the screen using Matlab and change the Matlab a little bit to get a closer look at the graphs"

These were some of the orientations that led to the establishment of a number of goals, some of which Sandy explicitly wrote down, and some we may infer. These include:

G1: To show students that interesting, unexpected things happen to the solution as the parameter changes.

G2: Students understanding, from the demonstration, that the solution of the difference equation approaches a periodic function.

G3: To show how easy it is to discover these things by using Matlab.

G4: To have students appreciate the value of Matlab as a mathematician's research tool. "But basically the use of computers to um.. explore mathematics that's something that I see as a mathematician and that I'd like to impart that.. I'd like the students to see that, the fact that they can learn a lot about mathematics using a computer." "The goal is to show students how we can solve problems with this technology and sometimes discover interesting mathematics in the process. We often try things that are not in the course book while not straying too far from it."

G5: To keep students interested. "…we wanted to keep them interested and so this was an extra lecture showing some more advanced features of the logistic equation that's usually taught at graduate level."

G6: To explain clearly the mathematical basis of the construction and solutions of the difference equation.

G7: To stick to the course book and cover all the assigned material.

To achieve these goals he called on resources, including his mathematical knowledge (R1) and the computer program Matlab (R2), which was used to plot solutions of the equation for various values of the parameter. The solutions were then displayed using an overhead projector (R3). At one point Sandy made the decision to move to the projected graph and show the students the periodicity of the function (see Figure 1a).

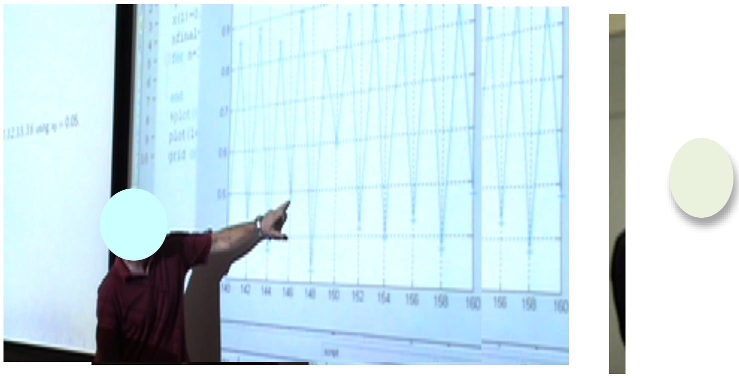

*Figure 1:* Stills from the video showing Sandy and Simon at critical decision points.

Why did he decide to do this? In line with his orientations O1, 2 and 5 it was part of his attainment of the goals G2, 3 and 6. O1 was crucial here, the belief that demonstrating and not just telling leads to understanding better, as he said "In fact the solutions are periodic and it was a bit hard to look and see that straight off that the solution's periodic so that's why I wanted to do the counting to count from one time step up until the time step when the solution repeats itself." However, it was during this process of counting the function local minima that a crucial decision point arose. The lecture transcription follows.

1. What's happening here it looks even more complicated, 3, 6…[*3 to 4 secs*] yeh so you can see that if you look at it closely…[*walks to screen*]
2 Suppose you start by looking at this value here [*pointing at the graph on the projection*] then there's going to be 1, 2, 3, 4, 5, 6, 7, you can count to 8 I think maybe.. do I ever get back to where I started, maybe not [*he realises that there is a problem*]
3 9, 10, 11, 12 ,13, 14 um.. how many values? So it looks like there's a period of um.. let's see 1, 2, 3, 4, 5, 6, 7, 8.. [*starts to count again*] it looks like there's a period of 14. Whether that's the case or not I'm not sure.
4 We might not have got to the limiting value yet. But it looks like we've settled down to a period of 14. By a period of 14 I mean that it takes 14 um.. we need $n$ to change by 14 to get back to where you started from. It looks like that.. something like that is happening anyway. So it seems to be settling down to some complicated periodic um.. solution.
5 All this.. all these things happen just by changing $q$. In fact if you do analyse this a bit further you can you can look at.. There are critical values of $q$ where you do get a change. And this allows you to draw what is called a bifurcation diagram. Bifurcation diagrams are diagrams that show how the solution splits up into different solutions as you change the parameter.

We see here that the count of the period arrives at 14, but Sandy knows that the true value is 16, as he later explained "…because the period doubles each time, so it goes from 2 to 4 to 8 to 16, so.. and so on, so there's a theory that actually says the period has to double." This discrepancy between 14 and 16 was unexpected, "I guess the thing that I was probably concerned about was um.. observing something that I didn't expect and not being about to explain it immediately". Hence, in-the-moment, he has to decide whether to address what is, for him, a mathematical discrepancy. How did he make the decision?

*Analysing the decision – Sandy*

Arriving at the decision involved an internal dialogue between the lecturer as a mathematician and as a teacher. This dialogue had as its aim the resolution of conflict between the competing pedagogical goals, G1, 2, 3 and 5, and the mathematician's goals of G4 and G6. We see this from Sandy's comments about this.

> Yeah, in fact my decision was based on the fact that I'd already spoken far longer than I'd planned to [G7, teacher] on the existing equation and it was time to actually go and do some problems [G7, teacher] which was supposed to be the rest of the lecture so I got onto that [G7, teacher].

> Actually I would have liked to have pursued it a bit [G6, mathematician] but we had already spent more than the allotted amount of time on this demonstration [G7, teacher] and I had shown them periodicity for shorter periods already [G2, teacher] so I think they had grasped the concept quite well so the fact that I didn't actually get a period of 16 bugged me a bit [G6, mathematician] but not enough to ruin the rest of the lecture [G7, teacher].

> I couldn't figure out at that particular time what the problem was [mathematician], uh.. I think I know now but even then it's not obvious… I think I actually probably could have shown them that it was periodic with period 16. [teacher–he has to be able to explain in detail]

> I certainly made a decision not to continue with an unexpected outcome on a graph in the first part of the lecture. Part of me wanted to address this at the time [G4, 6 the mathematician] but I had already gone over time with this part of the lecture and had achieved the goals I desired [G1, 2, 3 the teacher].

We see that, in *this* situation, with *these* students, the teacher wins out over the mathematician. The reason seems to be that the predominant goal was G2, to demonstrate that 'that the solution of the difference equation is a periodic function' and this had been accomplished, and the pedagogical goals were considered met, releasing him from the need to explain the mathematical anomaly, as Sandy said "Actually I didn't get any reaction from the students…I never did tell them it was really 16." He confirmed that, as a teacher, he was happy with the outcome of the lecture, including this decision:

> I'm pretty happy with the way the lecture went. Students seemed interested [G5, teacher], in our exploration of the logistic equation [O1, 2, 5; G1, 3, teacher] and participated in the exploration. In response to questions, we changed the Matlab code to zoom in on graphs of solutions, which allowed us to clearly see the periodicity [G2, teacher]. This part of the lecture occupied more time that I had anticipated [G7, teacher].

> The actual lecture itself went pretty well I was pleased with the demonstration [O1, 2, 5; G1, 2, 3 teacher] um.. I think it was clear enough. The students were able to see what was going on [G2, teacher].

*Simon – The mathematician takes control*

The primary purpose of Simon's lecture was to introduce the students to continued fractions. He too revealed a number of orientations that are relevant to this lecture and the decision we examine in detail.

O1 To emphasise to students that the right theoretical tools and proof techniques can tame a mathematical problem.

O2 Some proofs are more interesting and important than others. “The real reason I think it’s a cool proof is the fact that you prove a more general result. It’s one of these things that happens a lot in mathematics but you don’t see it so much at the junior level.”

O3 Mathematics needs to be correct. “Oh, this is not really right, I don’t like it not to be right.”

O4 Mathematical notation needs to be consistent and accurate. “Right so the symbols $h_j$ over $k_j$ will from now on will mean precisely one of these things for the specific numbers I am interested in.”

O5 Lecture notes need to be correct. “My response after this exercise was to go away and fix the notes because they were broken” and “it’s a tricky one because the notes clearly weren’t ideal because this whole thing was sort of hidden.”

O6 Lecture notes are useful and provide much of the detail. “I hand them out lecture notes at the beginning of each lecture with pretty much everything on them, they just have to fill in a few gaps.”

O7 Students who are talented at mathematics can cope when a lecturer dwells on the finer points in mathematics. “There is also a confidence that the students can cope with – if I go off on my own little journey the students will have the tools to deal with that … Whereas in another class you would be worried that if I’d lost them after 15 minutes then that’s it.”

O8 Some (but not all) all students at this level are ready to be inducted into mathematics. “Last year’s class I didn’t feel like they were being inducted into mathematics so it wasn’t necessary to dwell on this particular issue of the more general result.”

He also states the following beliefs about the class:

O9 The students are for the most part mathematically ‘strong’ and serious about learning mathematics: “They have a strong background in algebra and are able to work out details and read proofs in their own time.”

O10 There are 1 or 2 students who are not as quick as the others (but are still good and able students), and it is important for me not to go too quickly for them.

He discusses his role as a lecturer.

O11 Traditional lecturing by dictating notes is error prone

O12 My role as a lecturer is to provide a reason for coming to the lecture rather than reading it in a book. “I have something to add over the notes so it’s very much saying ‘Here’s the crucial bit. Here’s the important bit. Here’s the nice bit. Here’s the beautiful bit. Here’s the horrible dirty bit. This bit is really important, I should make sure I really know this bit. Or, yes this is meant to be a kind of tedious bit I shouldn’t feel bad about myself if I find this kind of boring. It’s to give them some sort of framework in their experience.”

In the ROG he writes before the lecture he states the following goals for the course:

G1 To help students to understand the theory and do proofs

G2 To provide good general preparation for post-graduate study in number theory.

G3 To increase the students’ mathematical maturity (is this about inducting them into mathematics?)

G4 To give exposure to different proof techniques.

In particular in this lecture he wants the students to learn a particular proof.

G5 The most important theoretical part is to state and prove correctness of the recurrence formulae for computing the convergents.

G6 “I hope to give a flavour for just how neat this proof is.”

Some of Simon’s goals were not stated in his written ROG but became apparent during discussion.

G7 To engage with the mathematics for its own sake, ‘it’s fun.’

G8 To ensure that the mathematics he does is ‘right’; it’s part of his role as a lecturer

G9 To use notation that is consistent.

G10 To induct (some) students into thinking and behaving like mathematicians.

G11 To present mathematics to his students in a way that gives them a “framework for their experience.”

In order to satisfy his goals he draws on a number of resources, including:

R1 His knowledge of mathematics in general and number theory, both theoretical and computational, in particular

R2 His assessment of the students’ mathematical ability and interest.

The decision we will discuss here is one he made when he suddenly realised that he was going to encounter a ‘notational conundrum’ while proving the correctness of the recurrence formulae for computing the convergents. [G5, teacher] in which he is aiming to ‘show them a very cool proof’ [G6, teacher] which would satisfy the course goals [G1, 2, 3 and 4, teacher]. When re-viewing the lecture he said, “It’s like I am labouring that point. This is the point [Transcript 2 below] where I have made a decision. I have somehow made a decision to somehow do it right.”[G8, mathematician] Which goals and orientations determine the outcome of the in-the-moment decision?

The lecture transcription follows.

1. This is the sum of $a_0$ plus, this thing is □□+*1*□□+*1* plus … [*indicates a succession of terms on board*] plus □□+*1*□□+*1*.
   So when you consider that separate extra term and we bundle that whole thing into one piece [*circles it on the board*] They are the same object.
   [*Stands back from the board and looks at class and then back at the board*]
2. So, by the inductive hypothesis, [*starts to write*] I know what this [*gestures in swirl over previous line*] It is some $h_i$ over $k_i$. [*looks at board*][*looks as if he is thinking*]
3. I’m going to call it … Did I give it a name? [*Looks at paper*]
   I didn’t give it a name. It’s just some $h_i$ over $k_i$. But whatever that $h_i$ over $k_i$ is it apparently satisfies the recurrence formula [*points to paper he is holding and looks at class*] [*Stands back and looks at the board*] [*Pauses a moment*] [*Moves*] [*Appears to come to decision*]
4. Yeah I mean this is *an* $h_i$ over $k_i$ but it’s not *the* $h_i$ over $k_i$ that I am really thinking of [*gestures back to previous expression*] This is a very subtle point

[*Said very quietly - thinking aloud?*]

5. [*Come back to board and writes above previous 2 expressions*]
Let's define $h_i$ over … Let's define $h_j$ over $k_j$ to be these things up to $a_j$ where I have worked these out already all the way up to *i* [*writes down and puts in rectangular box above previous 2 expressions*] *Right* so the symbols $h_i$ over $k_i$ will from now on mean precisely one of these things for the specific numbers I am interested in. This thing I have written down here [*gestures to bottom expression*] is *not* the $h_i$ over $k_i$ in that notation because this end term is wrong.

*Analysing the decision – Simon*

When we asked Simon why he thought he had made the decision to 'labour the point' and disentangle a problem of which the students were not (yet) aware and which "could only create confusion, because, dwelling on a fairly fine detail to such an extent might have obscured the bigger picture." He said it was really the mathematician within going "Oh this is not really right" [G8, mathematician] and added "at that point the whole world disappeared and it's just me and the mathematics." [G7, mathematician]

As he says in the interview, "This is exactly the point where I suddenly realise that it is sort of not quite fitting how I was using those symbols previously. I was thinking ahead to where the proof was going and suddenly it becomes clear to me that there is a problem ahead but it's not clear to anyone else yet." [R1] Visualising himself as the driver of a group of interested, but unworried, tourists (the students) who has just, to his surprise, noticed a sign saying 'Danger Ahead" on the roadside while the others "are all still happily chatting at the back of the bus" he says "I could put my foot down and hope." In that case he could have "swept it under the carpet," perhaps telling the students "It's in the notes … it would be a good exercise for you to do carefully." But he chose to sort it out, to labour the point, and take the bus down the bumpy road. [G7, mathematician and G2 and 10, teacher].

When he told the research group why he had chosen this section to re-view he spoke about ROG dissonance, a mismatch between what he did and the ROG that he had written before the lecture. Discussion uncovered higher order goals that drove the decision - his unwritten orientation and goals as a mathematician [G7,8,9, mathematician] and his desire to induct talented students into mathematics [G2,10,11, teacher]. Simon is not alone in needing to resolve this tension; Jaworski et al. (2009) also speak of the tensions experienced by lecturers arising from a desire to satisfy both student needs and mathematical values.

It was his estimation of students' ability [O9] that allowed him to take the 'detour'. His original assumption that because he saw them as good students he would say 'Just do it' proved incorrect in-the-moment. As he says 'last year there was not the same ability and I had a lower expectation of what they would understand and how they would work and so on so I must have gone through the proof much more lightly with them. Maybe it comes back to this inducting into being mathematicians. Last year's class I didn't feel like they were being inducted into mathematics so it wasn't necessary to dwell on this particular issue of the more general result.' [G10] The need to 'get it right' [G8,9] and to induct them [G10] won the day.

*Abi – The mathematician interrupts*

The lecture was an introduction to the Fibonacci sequence and the golden ratio for a general education first year course. In her personal ROG, written before the lecture, Abi stated the following, some of which we have classified as R, O or G:

R1 Knowledge of the mathematical subject material.

R2 Knowledge of the level of the students – and the mix of levels in the class: “A similarly difficult part of the class for me is and trying to keep everyone engaged and interested.”

O1 It is important to engage and intrigue students; “I have the class bring in pineapples, pinecones and sunflowers themselves – announcing in the previous lecture that they should do so with no hint of why – the idea being to make them think about why they would have to bring in a pineapple and hence create intrigue”.

O2 I see this whole course partially as an exercise in “public understanding of mathematics”, and so try to treat the lectures as such – rarely going into much depth mathematically.

O3 Mathematical enquiry begins with observation of the real world and we value that which we discover for themselves: “I give the class 5 or 10 minutes to try and count the spirals in each object. We then write down all the numbers we have seen in order and try to spot the pattern.”

O4 The physical counting is hard to do: “In the past this has caused some difficulties because it is actually quite hard to do, so this time I am going to bring electric tape to be used as a guide/marker for the spirals.”

O5 Prior knowledge of the students may be a problem: “Many of the students already know about Fibonacci and so announce the answer straight away, so I think there isn’t time for the students who don’t already know the answer to think about it.”

G1 The main aim is to give the students some idea of mathematical patterns that appear in nature, specifically patterns involving Fibonacci numbers.

G2 To keep the attention of the bright maths students whilst explaining the idea of a recurrence relation for the non-maths students.

G3 To get the students to spot the pattern(s).

G4 Rarely “going into much depth mathematically.”

G5 To help the students to calculate the value of the golden ratio.

From earlier discussions and her behaviour we were also able to infer the following belief:

O6 The mathematics we put in front of students needs to be correct.

Abi satisfies many of her goals by allowing time in class for the students to count the whorls of the pineapples and pinecones and providing the data to generate a table of the early Fibonacci numbers [O1, 3, 4 G1 teacher]. The students then provide a rule for the generation of the series based on their observations [G1, 3 teacher]. The lecture moves on to an examination of the ratio between successive terms. At this point we see an instance of the lecturer’s need for rigour. Note that nowhere in her written ROG did she mention rigour, on the contrary she says that she has a goal of “rarely going into much depth mathematically” [R1, G4 teacher/mathematician].

She is trying to establish in the students’ thinking that the limiting value for $f_{n+1}$ divided by $f_n$ is the golden ratio, $\psi$ [R1 mathematician]. There is some literal hand-waving as it is established by calculation that the value of □□+1□□ oscillates about 1.6 something [O2 teacher], and then she says:

1. OK suppose you want to compute what this number actually is.
   And it seems to be converging – **and it does actually converge** [*who is she reassuring*?]
2. So you know that $f_{n+1}$ is bigger than $f_n$ so this is going to be a number that is bigger than 1.
   Right? [*sounds as if she hears herself and adds this*] **Or equal to 1**.
3. So ..If I am thinking about what this ratio becomes as $n$ gets really, really big.
   **So, for any specific n these 2 things are going to be different.**
   Right?
4. Because for one thing it was 1.6 and for the next one it was 1.625.
   So for any specific $n$ it's going to be different.
   But as $n$ gets bigger and bigger and bigger these 2 things are going to get closer and closer together.
5. As long as $n$ is big enough. So we will assume that we are in a place where $n$ is big enough then **we can make this approximation.**

In this interlude we see and hear her spending a lot of time emphasising that for particular values of $n$ the values of $f_n$ and $f_{n+1}$ are different [R1 mathematician] and under what circumstances they are justified in making the approximation [O6, mathematician]. However, there is no need to establish the limit $\psi$ rigorously in this class [G4 teacher]. The bold highlighted language in this excerpt shows her need to be mathematically explicit in her terminology; hand waving will not do even in a class that is 'an exercise in public understanding of mathematics.' As the observer notes show (see Figure 2), later in the lecture Abi simply wanted to use the limit to calculate $\psi$ from the equation
□□+1□□= 1+ □□□□−1 without introducing the unnecessary concept of limit, other than to say "if $n$ is big enough".

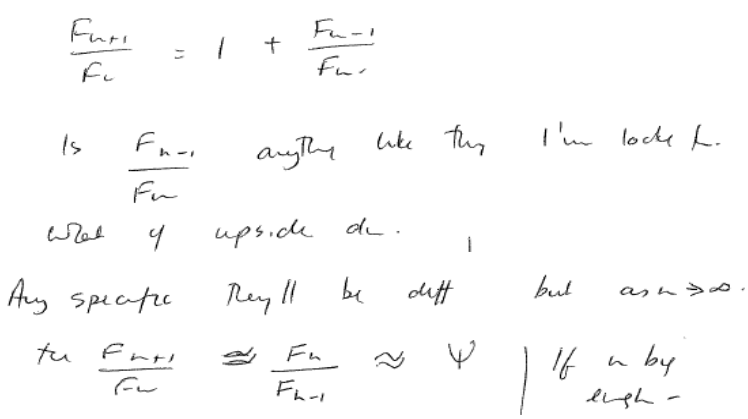

*Figure 2.* Observer notes from Abi's lecture showing the argument without use of limit.

In this instance it seems to us that the mathematician in the 'inner' discussion interrupts rather that taking over. Abi satisfies her need to be correct by adding qualifiers and conditions to the more general statements to ensure their mathematical correctness. This aligns with her remark during the earlier discussion but is at odds with her stated ROG prior to the lecture.

## DISCUSSION

We have described above three different scenarios and the resulting decision paths that arose, based on the inner discussion between the teacher and the mathematician. Group discussions have confirmed that such a dialogue is recognised as a reality, one that we have found is familiar to other lecturers as well. In our case, on one occasion Simon noted that:

> I do this kind of thing all the time, I think it's really distracting because you've gone out and tried to make your big point and then you get all flustered over some detail [of mathematics] and you say oh sorry you know, you have to get it right and the students go "what the hell is going on and now I'm completely confused because it sounded really simple."

To which Abi responded "So should you just ignore that corner [of important mathematics] and just hope that it's not noticed? But then is that bad because you've somehow told them something incorrect?" Our contention in this is that an awareness of this inner tension is an important part of reflecting on our role as tertiary teachers, which in turn will enable incremental growth in professional development, which Speer (2008) claims is an effective approach.

Is there any other value in discussing these examples? Do we have any evidence that this kind of discussion and analysis can aid lecturer professional development? Is a community of practice analysing in depth small parts of a lecture, against a framework of ROGs, in a manner similar to microteaching in schools, of pedagogical value? We believe that the feedback from all involved suggests that the answer to these questions is yes, and there are a number of important aspects contributing to this.

One is that having a 'mixed' community of practice of mathematics educators and mathematicians is important, enabling cross-fertilisation of ideas. For example, Sandy commented that "I'm happy with the discussion that the DATUM group had about the video clip. The analysis of the clip is encouraging in that I gained a mathematics education perspective of the clip, which clarified in my own mind what I do when I teach." and "…you come in with your mathematics education theory from time to time explaining some of the things that we all do and that's very useful as well. It's good to have some of the theory behind it." One aspect of the community that should not be underestimated was the opportunity to see others' teaching. Sandy thought that "And also seeing other people teaching, that's wonderful," and all agreed with a comment from Tim (another member of the group) that "not only is it useful for me and for feedback to mine but I actually find watching your lectures amazing." So much so that there was no shortage of volunteers to handle the video camera in others' lectures! Towards the end of one discussion session it was agreed that it would be a good thing if the practice of watching others became 'business as usual' in the department. This was confirmed in a comment from Abi that "When this project is finished it might be nice to somehow have some sort of setup where lecturers are paired up or in threes or whatever and regularly go to each others' lectures." Thus she proposed continuing some form of supportive departmental community after the conclusion of the research. While we agree that this would be useful, we contend that the subsequent, focussed discussion is also extremely important.

As second aspect is that the community was supportive. Sandy's thoughts on this were that "It was quite reassuring that nobody thought that [he looked silly]. Yes, so that was good."
and that the group was "Oh yes, very supportive, very supportive." Simon's comment,

"It's pretty revealing watching yourself being videoed isn't it?" shows that lecturers were sensitive about exposing their practice. However he also stated "I would like to sort of record that I have been very happy with the supportive atmosphere, it's been very.. I haven't felt nervous about having people in my lecture or watching the video." It is worth noting that none of the participants was reluctant to repeat the process. From a practical, teaching perspective it was thought that feedback and discussion was important. Sandy's view was "So that's one of the nice things about having these um.. these discussions and um.. about the videos is getting feedback from people." He also mentioned "So it's good to get that feedback from other people and in some cases people identify things that I do that I wasn't even aware of." and "It's good to have feedback on that. Because like other people I guess I have my usual techniques for teaching and um.. but it's good to get some opinions on these techniques from other people." The discussions called Simon's attention to how his decision was predicated on the assumption that this was a 'better' class – prompting him to say "anything that encourages me personally, to put more thought into who the audience really are, what actually they know. That's extremely useful."

The development of a community of practice was further evidenced by the fact that when we were viewing Simon's lecture we observed that "you looked how [Sandy] looked when he was worried about the thing." No-one needed elaboration of 'the thing,' we all knew it referred to his dilemma and subsequent decision. As we work the repertoire of shared decision moments is growing and enables new ones to feed off them.

Apart from the discussions, the process of thinking about, and then writing, one's ROG was another feature commented on by several community members. Sandy said how "It just clarified what I planned to do. I think it may have affected what I did, what I was going to do…It was good to have a clearer idea of what the goals were before going in. It probably helped to make the lecture, well to improve the lecture. I'm sure it did." In Simon's case the 'dissonance' he perceived between his stated ROG and his decision, led to a discussion of his higher order goals. This discussion about his decision increased his awareness of how his decision was predicated on the assumption that this was a 'better' class – and led him to say that he would like to think more about the audience. "Before a lecture you should sit down and remind yourself who is in the class, what they know, and it's astonishing to think that that is not automatically done." Tim said that "I actually found the writing of the ROG quite a useful thing to do…It made me.. in some ways that made me think more than I ever have about what I'm doing in the lecture." Simon agreed with this:

> I agree with [Tim] that writing the ROG was a very.. I think a very useful thing to do and I wish I could hold on to that better. I think.. It forced me to think much more clearly about what I.. what my expectations of the students really are…I think I probably should write a ROG every time five minutes before my lecture just for the pure reason to remind myself that there are students out there and they have a position.

The ROG structure also provides a framework for discussing the 'objects of practice' the lecturers chose – similar to the manner in which Simon says he want to provide a mathematical framework for his maths students. Engaging mathematicians (even more than teachers) in a conversations about pedagogy – particularly their own – is enabled by linguistic and theoretical support that is grounded in their practice.

It appears that the fine-grained analysis, against the framework of the ROGs activates an awareness of the basis on which we make teaching decisions and prompts examination of these decisions leading to development of practice. In addition the analysis adds to what we know about professional development at collegiate level and the influence of beliefs on teaching practices "at the very level of detail where it appears development most *productively occurs*" (Speer, 2008, p. 219, italics original) and informs practice. Along with Jaworski et al. (2009) we argue that knowing more about the practices we, and our colleagues, engage in will enable us to develop them in informed ways. There is a nice parallel with the practice in mathematics of using and interrogating the specific, special case to begin to build an understanding of the general.

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