Promoting Success in College Algebra by Using Worked Examples in Weekly Active Group Sessions

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Abstract: At a research university near the east coast, researchers have restructured a College Algebra course by formatting the course into two large lectures a week, an active recitation size laboratory class once a week, and an extra day devoted to active group work called Supplemental Practice (SP). SP was added as an extra day of class where the SP leader has students to work in groups on a worksheet of examples and problems, based off of worked example research, that were covered in the previous week's class material. Two sections of the course were randomly chosen to be the experimental group and the other section was the control group. The experimental group was given the SP worksheets and the control group was given a question and answer session. The experimental group significantly outperformed the control on a variety of components in the course, particularly when prior knowledge is factored in.

Keywords: College Algebra, Cognitive Science, Worked Examples, Large Lecture, Supplemental Sessions

Introduction

A Commitment to America's Future: Responding to the Crisis in Mathematics and Science Education states that "nationally 22% of all college freshman fail to meet the performance levels required for entry level mathematics courses and must begin their college experience in remedial courses" (2005, p. 6). The enrollment in College Algebra has grown recently to the point that nationally there are estimated 650,000 to 750,000 students per year (Haver, 2007) and has surpassed the enrollment in Calculus. Although there are almost three fourths of 1 million students enrolling in College Algebra, it is estimated conservatively that 45% of these students fail to receive a grade of A, B, or C and can reach percentages in the sixties at some colleges. To address this non-success of students at a large research university in the eastern part of the United States, faculty members teaching College Algebra have implemented a new structure in the course that emphasizes active learning through a day called Supplemental Practice.

Theoretical Framework

The Interactive, Compensatory Model of Learning (ICML) provides the framework for understanding and improving classroom learning (see figure 1). Schraw and Brooks (1999) refer to a wide range of literature that reinforces ICML. There are five main components of ICML: cognitive ability, knowledge, metacognition, strategies, and motivation, which affect learning. The following are brief definitions of the five main components of ICML found in (Schraw and Brooks, 1999) and readers should reference their work for more detailed information. *Cognitive Ability* refers to one's general capacity to learn and *knowledge* refers to organized domain-specific and general knowledge in long-term memory. *Metacognition* includes knowledge about oneself as a learner and how to regulate one's learning. *Strategies* refer to procedures that enable learners to solve specific problems and *motivation* refers to beliefs about one's ability to successfully perform a task, as well as one's goals for performing a task.

Figure 1 shows that knowledge, metacognition, and strategies are so closely connected that they are combined together into one area in the figure. We will refer to this one area as knowledge-regulation component. The ICML captures the interactions between these four components that affect learning and describes how one component can compensate for deficiencies in others.

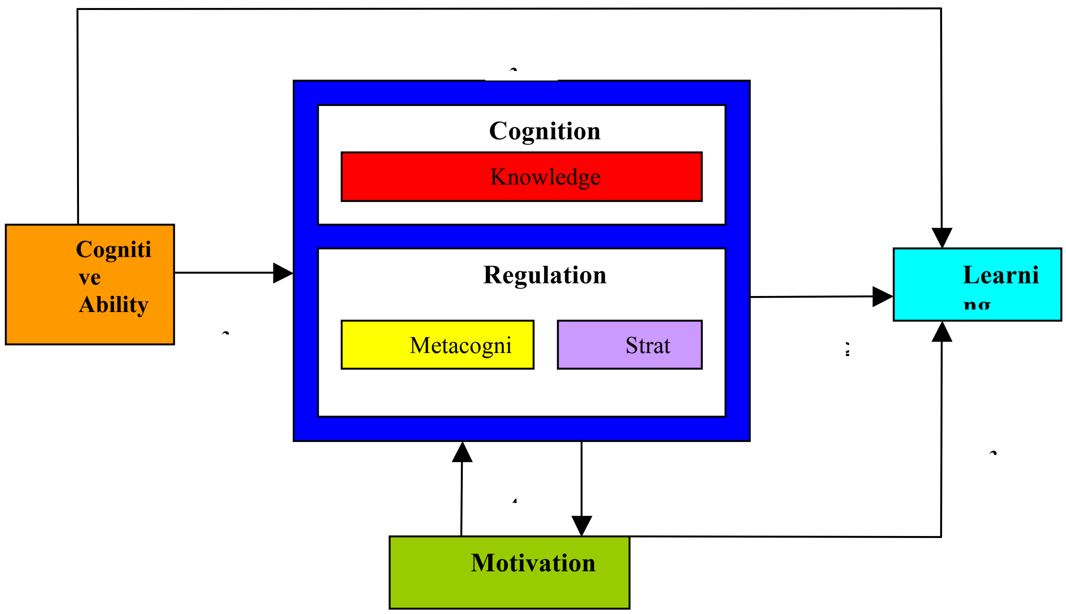

Figure 1: Interactive, Compensatory Model of Learning

Each component can affect learning either directly or indirectly. For example, cognitive ability is related to learning directly, but also indirectly through knowledge-regulation. From Figure 1, one can see that each component directly affect learning (the arrows from each component to learning) while only some components affects learning indirectly through another component. For example, motivation indirectly affects learning through knowledge-regulation, but not through cognitive ability. The numbers in the figure refer to the estimated correlation coefficient between two components. Each correlation coefficient is the estimated value of what has been measured in a number of empirical studies. Cognitive ability is correlated to learning with correlation coefficients ranging from 0.3 to 0.4 (Brody, 1992) and hence, the correlation coefficient of 0.3 between these components. The other correlation coefficients are shown on Figure 1. Schraw and Brooks (1999) state that "most experts agree that knowledge and regulation exert a strong direct effect on learning that is greater than the effects of either ability or motivational beliefs" (p. 9).

The compensatory part of the model refers to how students can compensate for a weakness in one component with a stronger component. For example, students who have weaker cognitive abilities (literature refers to this as intelligence) can compensate by a stronger knowledge-regulation component. That is, students can regulate their learning while they work diligently to increase their knowledge about a particular topic. Through this iterative process, as they go from one topic to another topic in the course to gain knowledge, they successful compensate for their lower cognitive ability than other students. The notion of compensatory processes is supported by many different theories (Gardner, 1983; Perkins, 1987: Sternberg, 1994). Schraw and Brooks (1999) state the following compensation can occur:

1) ability compensates in part for knowledge and regulation
2) knowledge and regulation compensate for cognitive ability and motivation, and
3) motivation compensates for ability, knowledge and regulation.

The next section discusses the history of supplemental practice and briefly covers literature on worked examples.

Background and Literature Review

Supplemental Practice Structure

The idea of Supplemental Practice, denoted SP, was implemented during the Fall 2004 and was originally modeled after Supplemental Instruction (Arendale, 1994; SI Staff, 1997). The normal structure of the algebra class that consisted of three lectures a week morphed into a structure of two lectures a week in a large lecture room, and an active laboratory class once a week in computer classrooms where students meet in smaller groups. The lab class was held on Tuesdays while the lecture class was held on Mondays and Fridays. The SP days on Wednesdays were originally added to the schedule to help lower-achieving students. This was done by requiring students that scored lower than an 80 on a placement exam, or scored lower than a 70 on any regular exam, to attend the SP sessions. Starting in the Fall 2006 semester, the SP sessions have since morphed into active problem-session days modeled after the cognitive science "worked-out example" research. The worked-out example research, henceforth denoted worked examples, asks students to study a worked example for a particular topic, ask questions about anything in the example that they do not understand, and finally work a similar example without reference to the worked example, nor other outside sources (Cooper and Sweller, 1985; Ward and Sweller, 1990; Zhu and Simon, 1987; Carroll, 1994, Tarmizi and Sweller, 1988). The SP sessions and worksheets have been developed based off of the worked example research for the following reasons: 1) helps students to be actively engaged with the material in a setting where they can get feedback and assistance as they solve problems, 2) assists students in transferring information from working (short-term) memory to long-term memory, 3) helps students to regulate their learning and build confidence that they can work problems, 4) allows the instructor to work in the large lecture class to assist students as they learn the material, and 5) helps students as they work on homework, quizzes, and while they study for tests outside of class.

Worked Example Research

The discipline of cognitive science deals with the mental processes of learning, memory, and problem solving. Worked example research was developed based from Seller's cognitive load theory (1988). The total load on working memory at any moment in time is referred as the cognitive load. Most people can retain about seven "chunks" of information in their working memory and when they exceed that limit at any moment in time, there will be a loss of information in the working memory. In other words, there is an overflow of information in the working memory and cognitive overload. Cognitive overload can be thwarted if one limits information so that it does not exceed the students' working memory. One way this can be done is to transfer information from working memory to long-term memory as information is being processed (or soon after). According to Sweller (1988), optimum learning occurs in humans when one minimizes the load on working memory, which in turn facilitates changes in long-term memory.

Worked example research (Cooper and Sweller, 1985; Ward and Sweller, 1990; Zhu and Simon, 1987; Carroll, 1994; Tarmizi and Sweller, 1988) present students with a worked example on paper and tells them to study the example. Once the students are done studying the worked example and have asked any questions, the instructor asks the student to solve a similar problem without any help from the worked example. It has been suggested that worked examples reduce the cognitive load on a student and might optimize schema acquisition (Sweller and Owen, 1989**;** Sweller and Cooper, 1985). In addition, worked examples have been researched (and used) in a variety of subjects: mathematics (Cooper and Sweller, 1985; Zhu and Simon, 1987), engineering (Chi et al., 1989), physics (Ward and Sweller, 1990), computer science (Catrambone and Yuasa, 2006), chemistry (Crippen and Boyd, 2007), and education (Hilbert, Schworm, and Renkl, 2004**).** Readers can reference these studies to gain more information and insight. We will mention highlights of a few of these to end this section.

Sweller and Cooper (1985) conducted one of the first studies on worked examples in high school-level Algebra. Through five experiments, they examined the use of worked examples as a substitute for problem solving and consistently found that the worked example group outperformed the conventional group (worked conventional homework problems instead of studying worked examples) and had less time on task. Zhu and Simon (1987) demonstrated the feasibility and effectiveness of teaching mathematical skills through chosen sequences of worked examples and problems in a Chinese middle school's Algebra and Geometry curriculum without lectures or other direct instruction. Chi et. al. (1989) showed that while students studied worked examples, "good" students generally monitored their own understanding and misunderstanding through self-explanations. Compare this to "poor" students who did not generate sufficient self-explanations or monitored their learning inaccurately. Ward and Sweller (1990) established that students who used worked examples formatted to reduce the need for students to mentally integrate multiple sources of information achieved test performances superior to either those exposed to conventional problems or to those shown worked examples that required students to split their attention.

Past research on worked examples in mathematics has been conducted in a laboratory setting. This research is conducted in a large lecture classroom setting and concentrates on determining if worked examples helped promote success in the course. In addition, past worked example research in mathematics has not dealt with college mathematics courses, classes in a large lecture setting, or implementing an extra day of class to focus on working with students to master material. The research could be valuable to other researchers that are working to promote student success in large lecture classes. The research question that will be addressed in this study is "Do students in the experimental group earn significantly different course grades/exam scores/quiz scores/etc… than students in the control group?"

Prior Data

Data has been generated for all students in College Algebra, including SP sessions attended, since Spring 2007. Students historically perform better in the class as the number of SP days they attended increases and the success rates – the number of students that receive a specific grade, usually a D or C, divided by the total number of students – reach the 60 – 80% range for students that attended 9 or more SP days in a given semester. Figure 1 shows success rates versus number of SP days attended in the Spring 2007.

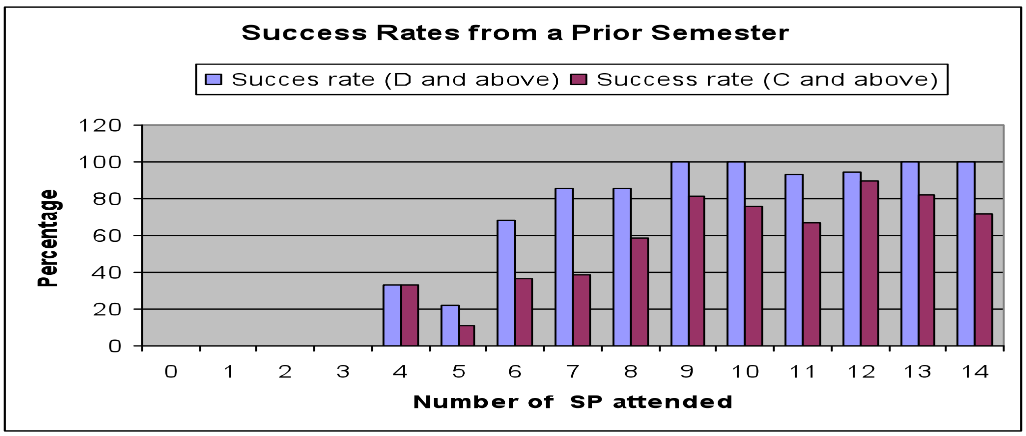

Figure 2

The overall course averages in the spring 2007 College Algebra course is shown in Table 1 below. We see that students that attended 8 or more SP days earned overall course averages in the 70 percentile range with a maximum course average of 77.8% for students that attended 12 SP days. Also all, but the students who attend 8 and 11 SP days earned course averages of 75% and above. The data looks very promising in showing that SP help students in the course. However, the data could be suspect because students voluntarily attended the supplement sessions and perhaps only the motivated students attended the majority of the sessions. To be able to prove that SP sessions were beneficial to the students, the author designed an experiment prior to the Fall 2009 semester to give the experimental group the treatment (worked example worksheets) and the control group an alternative session.

# of Supplemental days attended	Course Average	# of students
0	10.615	2
1	16.185	10

2	22.615	4
3	20.893	3
4	35.630	3
5	45.078	9
6	63.376	22
7	67.040	54
8	71.746	34
9	75.354	27
10	74.963	29
11	73.771	15
12	77.838	19
13	76.625	11
14	75.579	7

Table 1

Methodology

Course

The setting for the research was a large lecture 4-day College Algebra course with an annual enrollment of around 1000 students. This course is one of three different types of College Algebra courses at the university. One type of College Algebra is called a 3-day large lecture College Algebra course that comprises of two lectures a week in a large lecture setting and one day a week in the lab where students actively work in smaller-group math labs. The second type is the 4-day College Algebra course which has the same format as the 3-day College Algebra course, except the 4th day is spent in SP. The final type is a 5-day College Algebra course that is comprised of 5 lectures a week in a class size of approximately 40 students. The 5-day College Algebra class takes all their quizzes and exams by pencil and paper and does not have a laboratory component. All College Algebra courses require specific placement exams scores. The 3-day College Algebra course requires the highest placement score and the 5-day College Algebra course requiring the lowest placement score. In addition, students in the 3-day and 4-day large lecture College Algebra courses take their computerized exams during lab and their quizzes at home on a computer. The pencil and paper labs are completed during lab time one day a week with the help of a java-based applet grapher.

Worked Example Worksheets

During the SP days, worked example worksheets were handed out to the students to work on in groups. SP days were moved from Wednesdays to Mondays in Fall 2008 so that students would have SP days before the exams that occurred periodically on Tuesdays. Since the class was still in the large lecture classroom setting with theatre-seating structure, students formed groups with other students near them as they see fit. Usually students worked with 1 to 3 other students seated close to them. The worked example worksheets consisted of an expert solution of a College Algebra problem followed by a problem for the students to work out. For example, the following worked examples (see figure 3) were given on worksheet 3 (left column) and 11 (right column) during the 5th and 13th SP sessions (the first SP occurs on the third Monday of the semester). These two problems are stated exactly like they are on the worksheet. The worksheet is always given to the students as one sheet (front and back) in a two column format with headings on all worked examples, followed by the section in the textbook (Sullivan and Sullivan, 2006) that can be referenced later outside of class. There are approximately 8 to 10 worked examples and problems on each worksheet. The material on the worksheets consisted of some of the material covered during the previous week in lecture (too much material to

Worked-out Example (2.2):

Find the slope and y-intercept for the equation $2y+3x=1$.

To find the slope and y-intercept we need to rewrite the equation into the form $y=mx+b$. That is, solve for y. Thus rewriting $2y+3x=1$ we have $2y=-3x+1$ which simplifies to . So the slope is $-\frac{3}{2}$ and y-intercept is $\left(0,\frac{1}{2}\right)$.

2(b) Find the slope and y-intercept for $\frac{1}{2}y+5x=-3$.

Worked-out Example (6.6):

Solve for x.

Using the property $\log_b x+\log_b y=\log_b(xy)$, we have . Changing this from logarithm to exponential form, we have $4^1=x(x-3)$. So

$$4^1=x^2-3x$$
$$0=x^2-3x-4$$
$$0=(x-4)(x+1).$$

So or . Therefore $x=4$ or $x=-1$. But we can not include $x=-1$ as a solution because when we substitute it back into the original equation it yields $\log(-1)$ and $\log(-4)$ which are undefined since we can not take a logarithm of a negative number. Solution is $x=4$.

1. Solve $\ln(x+1)-\ln x=2$ for x.

Figure 3: Sample Worked-out Examples from a Worked-out Example Worksheet

cover it all). No new material was ever covered and the worksheets comprised of problems directly from or derived from the problems in the textbook. The worksheets were never developed while referencing material from exams, quizzes, or labs. However, most of the questions from the exams and quizzes are similar to the homework in the book. Finally, the worksheets are modeled after worked example research since it presents an expert's solution to a problem followed by a problem for the student to work out. The only difference is that it is not plausible to ask the students to not reference the worked example while working another problem and so this was never done. Furthermore, most studies on worked examples state that the student should be given a similar problem (very similar in some cases), but in SP the problems students were asked to do, vary from very similar to somewhat different problems.

Experiment

The researcher randomly designated one of the course sections as the control group (n = 177) and the other two sections as the experimental group (n = 320). In the experimental group, the students were given a "worked-out example" worksheet at the beginning of each of the 13 SP days and asked to work in groups to complete the worksheet. Three to four class assistants circulated around the room to answer any student questions about the worksheet. In the control group, a graduate student organized a question-and-answer session during the extra day instead of giving a worksheet to the students. Students were able to get any question answered, but the graduate student only answered student questions and did not generate questions themselves. For the most part, the graduate student spent most of the class time answering student-generated questions. Quantitative data (course scores on exams and quizzes, supplemental days attended, class attendance, total points,…) was collected for each student in both the control and experimental groups and analyzed at the end of the semester. There were similar demographics in both the control and experimental groups.

Data

Data from the experimental and control groups were compared on a variety of levels by using t-test with equal and unequal variances depending on the data. The experimental and control group had similar levels of retention (number of students that completed the course) at 80.5% and 84%, respectively. At the beginning of the semester, all students were given an old ACT math test that consisted of 60 questions. Students were given extra credit points for the ACT test on a sliding scale. This ensured that the better a student performed, the more extra credit, up to 10 points, they earned. The ACT exam gave a good measure of students' prior mathematical knowledge. Figure 4 shows the control and experimental groups' mean scores of 28.40 and 26.91 with standard deviations of 6.41 and 6.86, respectively. The control group significantly outperformed the experimental group (p=0.01) on the (pre) ACT test.

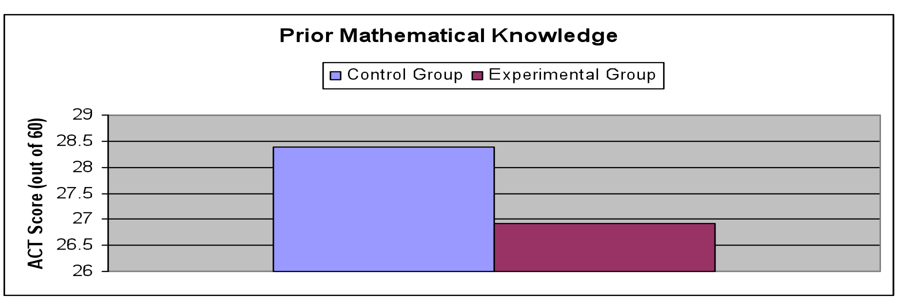

Figure 4

At the end of the semester, students were given the same old ACT exam to measure their post mathematical knowledge. Figure 5 shows the control and experimental groups earned a mean ACT score of 32.81 and 32.16, with standard deviation of 6.46 and 7.22, respectively. There was no significant difference between the mean ACT scores of the two groups.

The data for total points in the course (Current Points), total points without attendance (CP w/o Attend), total points without attendance or labs (CP w/o Attend, Labs), and Current points for just exams (CP Tests Only), were compared between the three groups.

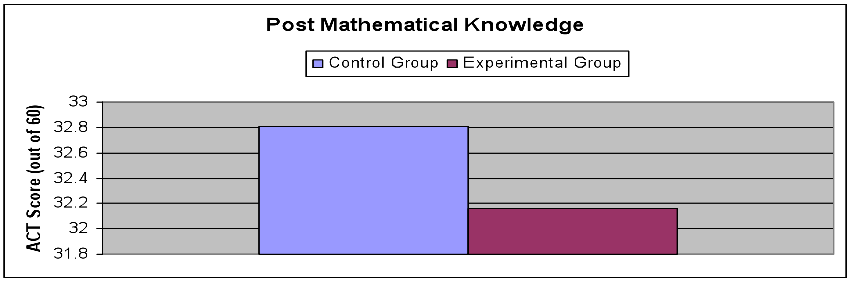

Figure 5

Figure 6 shows the current points for the two groups and Table 2 shows the mean current points with standard deviations.

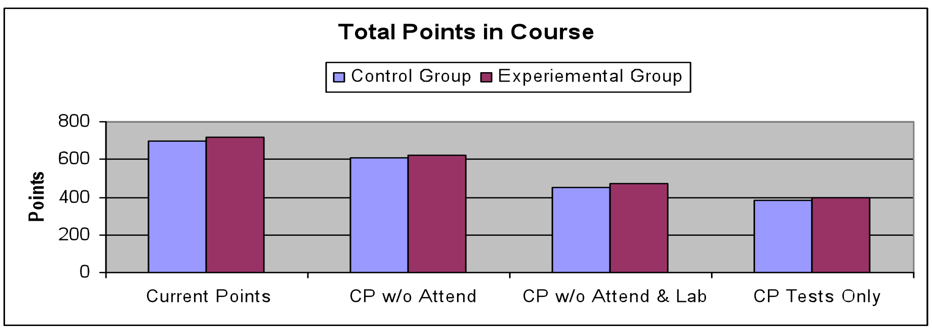

Figure 6

	Current Points	CP w/o Attend	CP w/o Attend, Lab	CP Tests Only
Control Group (n=177)	698.81 (150.51)	605.88 (140.25)	453.24 (120.04)	381.72 (105.48)
Experimental Group (n=320)	718.60 (151.60)	625.37 (141.29)	470.82 (119.49)	396.25 (103.98)

Table 2: Means and standard deviations

There were almost significant differences between the mean scores of the control and experimental groups with respect to Current Points ($p = 0.082$), CP w/o Attend ($p = 0.070$), and CP Tests Only ($p = 0.069$). The experimental group had significantly better current points without including the attendance and laboratory points than the control group ($p = 0.058$). Recall that current points did not include any extra credit (i.e. pre/post ACT exam).

The two groups were compared with respect to each exam, the final, and quizzes. Figure 7 shows the two groups mean scores on each exam, the final, and quizzes and Table 3 shows the exact scores and standard deviation in parentheses.

The experimental group significantly outperformed the control group on test 3 ($p = 0.031$), quizzes ($p =0.048$), and final exam ($p = 0.029$), and almost significantly outperformed the control group on test 2 ($p = 0.083$). There was no difference between the control and experimental groups with respect to test 1 or test 4.

Course grade point average was calculated to compare the two groups on the average course grade earned. This was accomplished by assigned a quantitative score for the final grade that each student earned in the course (A = 4, B = 3, C = 2, D = 1, and F = 0). Figure 8 shows the course grade point average for control group (1.97) and experimental group (2.13) with standard deviations 1.17 and 1.27, respectively. The experimental group had an almost significantly different course grade point average than the control group ($p = 0.075$).

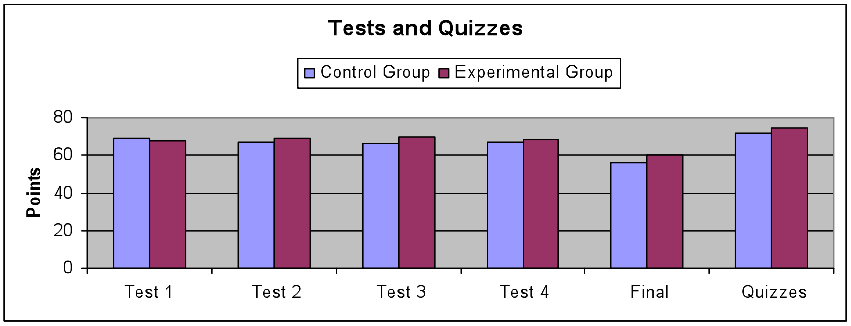

Figure 7

	Test 1	Test 2	Test 3	Test 4	Final	Quizzes
Control Group (n=177)	68.84 (16.21)	66.86 (19.93)	66.58 (19.61)	67.23 (22.67)	112.20 ()	71.52 (18.89)
Experimental Group (n=320)	67.52 (16.18)	69.38 (17.99)	70.05 (19.76)	68.66 (23.88)	120.66 ()	74.57 (20.65)

Table 3

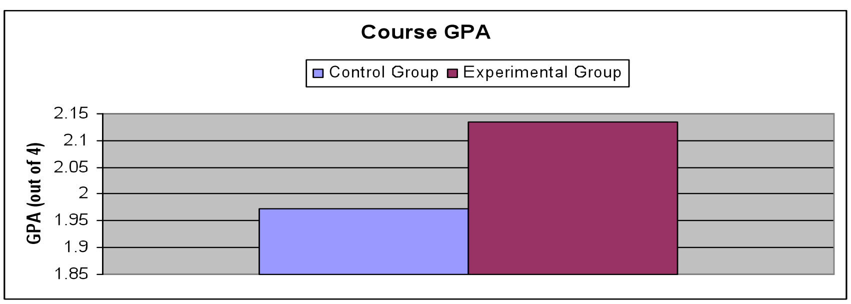

Figure 8

Experimental Group versus Control Group Based on Levels of Prior Knowledge

The ICML states that students can compensate for weaker prior knowledge by having a stronger knowledge-regulation and motivation component. To investigate this, the researcher examined the difference (significant or not) between the control and experimental groups based on levels of prior knowledge. The high prior mathematical knowledge group was defined to be all students with a 31 or higher score (out of 60) on the old ACT math exam. The high prior mathematical knowledge control ($n = 54$) and experimental group ($n = 88$) was denoted as high control and high experimental. The medium prior mathematical knowledge group was defined to be all students with an old ACT math exam score of 26 to 30. The middle prior mathematical knowledge control (n = 59) and experimental group (n = 89) was denoted as middle control and middle experimental. The low prior mathematical knowledge group was defined to be all students with an old ACT math exam score of 25 or below. The low prior mathematical knowledge control (n = 49) and experimental group (n = 119) was denoted as low control and low experimental.

Figure 9 shows the prior/post mathematical knowledge for the control and experimental group based on the level of prior knowledge groups. For prior and post, there was no significant difference in mathematical knowledge for the high control (pre = 35.26, post = 37.21) and high experimental (34.84, 37.36), nor the low control (21.14, 28.74) and low experimental (20.45, 27.82) with standard deviations (4.02,5.41), (3.74, 5.24), (3.25, 5.47), and (4.07, 6.72). However for the prior, the middle control (28.14) significantly outperformed the middle experimental (27.71) (p = 0.041) with standard deviations of (1.48, 5.30) and (1.43, 5.71), respectively. For the post, there was no significant difference in the middle groups.

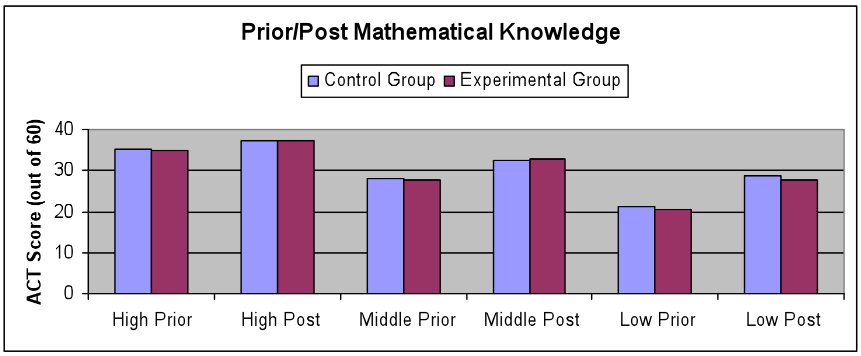

Figure 9

Furthermore, there is no significant difference between the performance of the high control group and the high experimental except on test 3. The high experimental (78.13) almost significantly outperformed the high control (74.17) on test 3 (p = 0.065) with standard deviations 15.34 and 14.59, respectively. Similarly, there is no significant difference between the performance of the low control and low experimental except on test 1 and test 2. The low control (64.80) almost significantly outperformed the low experimental (61.30) on test 1 (p = 0.085) with standard deviations of 14.25 and 16.42, respectively. In addition, the low experimental (63.91) significantly outperformed the low control (59.08) on test 2 (p = 0.051) with standard deviations of 19.71 and 16.06, respectively. The middle prior mathematical knowledge level is where the significant data emerged. The middle experimental significantly or almost significantly (two cases) outperformed the middle control on every course component except test 1. See Table 4, Figure 10, and Figure 11 for more details.

	Mean Control (n=59)	Mean Experimental (n=89)	Standard Deviation Control	Standard Deviation Experimental	P – Value
Current Points	669.18	726.58	175.70	140.70	0.019
CP w/o	578.58	632.09	160.41	130.56	0.017

Attend					
CP w/o Attend, Lab	431.33	474.89	131.46	110.62	0.019
CP Tests Only	362.12	398.65	114.91	95.93	0.023
Test 1	67.88	69.66	15.18	13.87	0.235
Test 2	64.41	69.49	22.32	15.45	0.065
Test 3	66.19	72.53	19.94	17.44	0.025
Test 4	63.47	69.33	24.48	24.54	0.079
Final	100.17	117.64	54.63	45.90	0.023
Quizzes	69.21	76.24	20.09	20.43	0.020
Course GPA	1.85	2.19	1.26	1.22	0.050

Table 4: Middle Prior Knowledge Mathematical Level

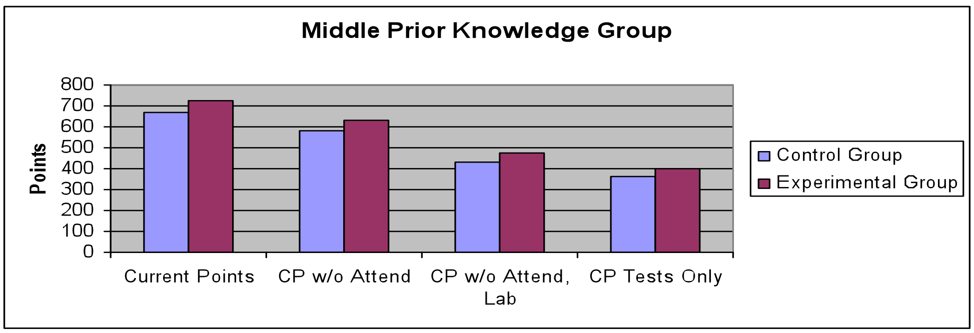

Figure 10

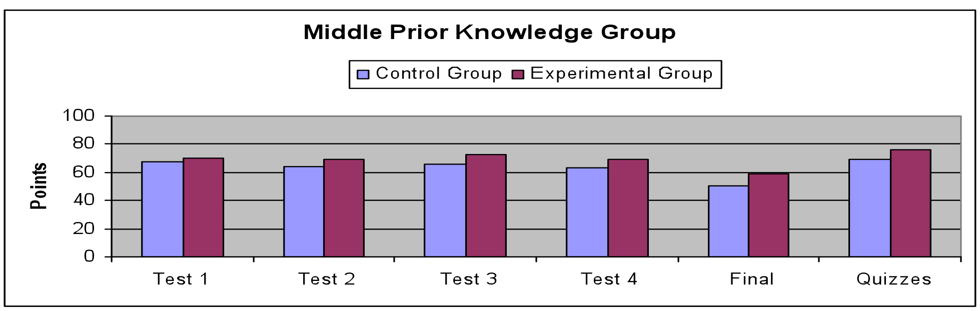

Figure 11

Results

For the most part, prior data has shown that the more days of supplemental practice that a student attends, the better the student performs overall in College Algebra. Specifically, students who attend 9 or more SP days have been very successful in earning a grade of C or better in the class. In this past data that has been collected, students voluntarily attended the SP sessions on Mondays. One could argue that the students that were motivated came to the SP sessions 9 or more times and hence that is why they were successful. This research study was developed to investigate whether the SP sessions help students to be more successful on course components and overall.

We see that the control group started with a significantly higher prior mathematical knowledge than the experimental group. However, the control and experimental groups ended the semester with no significant difference in post mathematical knowledge. The students in the experimental group added to their prior knowledge throughout the class so that they had similar post mathematical knowledge, assessed by an old ACT exam. On average students in the experimental group increased their ACT exam score by 5.15 points compared to an increase of 4.32 points for the control group. This is very important since ICML states that students can compensate a weaker knowledge component with a stronger motivation component. Examining things a little further, we see that the high and low level prior mathematical knowledge groups started off with similar prior knowledge but the middle prior mathematical knowledge control group was significantly higher than the middle prior mathematical knowledge experimental group. However, all three experimental groups (high, middle, and low) ended the semester with similar post mathematical knowledge as the three control groups. The middle group started with significantly lower prior knowledge, but ended with similar post knowledge in the course. We can conclude that it was the middle group's mathematical knowledge that was affected the most by the worked examples. The worked examples worksheets helped the students in the middle experimental group to accumulate the knowledge needed to bring their mathematical knowledge to a similar level of the middle control group.

In terms of total points in the course, although the experimental group did not always significantly outperform the control group, the experimental group almost significantly outperformed the control group. When attendance and laboratory components of the grade were removed from the current points, the experimental group significantly outperformed the control group. Investigating things further, we find that, for the most part, the middle prior mathematical knowledge experimental group significantly outperformed the middle prior mathematical knowledge control group. In fact, there were only three components where no significant difference or almost significance occurred. The non-significance on exam 1 for the middle groups is understandable since there were only two SP days before exam 1 and students had to get comfortable with the worked example worksheets and the structure of the SP sessions. Both exam 2 and exam 4 performances showed that the middle experimental group outperformed the middle control by a half letter grade and was very close to being significant.

In conclusion, we find that the SP sessions benefited the middle experimental the most out of all other groups. Using the ICML framework, SP sessions helped the students in the middle experimental gain a stronger motivation component to compensate for a weaker knowledge component. In addition, as their stronger motivation compensated for weaker knowledge, they increased (strengthened) their knowledge through learning in an iterative process throughout the semester. This led to more learning directly from their knowledge-regulation component and indirectly through motivation. We propose that the high experimental was not motivated any differently than the high control. That is, the high prior mathematical knowledge group would learn the material no matter what intervention was given in the class. According to ICML, there are two components that can compensate for the knowledge-regulation component: ability and motivation. It is not completely clear why there was no significant difference in the lower prior mathematical knowledge groups. The research by Chi (1989) might shed some light, perhaps the high prior mathematical knowledge group is comprised of the "good" students that have sufficient self-regulation skills and the low prior mathematical knowledge group is comprised of the "poor" students that do not have sufficient self-regulation skills. We propose as a whole that the lower groups start the course with such a low prior knowledge-base that they are not able to compensate for this lack of knowledge through cognitive ability or motivation. It would be interesting to see if the higher cognitive ability students in the low prior mathematical knowledge group are able to compensate for the weak knowledge-regulation component. This would illuminate whether cognitive ability compensates for knowledge-regulation for the lower prior mathematical knowledge group. Future research will attempt to investigate this and why the middle group seems to benefit the most from motivation. It is important to note that data has been analyzed more for the experimental group based on their attendance of SP days. It turns out that the students in the experimental group that attend 8 or more SP days outperform the control group significantly on every comparison (p-values less than 0.0091 expect a p-value of 0.047 for test 4). The results are even more significant when we compare the students in the experimental group that attended 13 SP days to the control group. In fact, this group significantly outperforms any other group of students with a specific number of SP days attended. For example, the group of students in the experimental group that attended 13 days of SP significantly outperform the group of students in the experimental group that attend 12 days of SP. Details of this research will be presented in future papers.

Implications to Teaching

Many college instructors teach large lecture sections of introductory mathematics classes and struggle with high percentages of students that earn grades of D or F, or simply withdraw (DFW rate) from the class. It takes resources to offer recitation sessions, out-of-class sessions, or tutoring. These are familiar modes of intervention that colleges use to lower the DFW rate and, in general, help students be more successful in learning material in a course. This study shows that carefully designed worksheets modeled after worked examples coupled with active group sessions can be very beneficial in helping students become more successful despite lower prior knowledge in a course. The SP day each week allow extra active session where students can work on comprehending material in groups. These SP days are like an extra day in class, however, they only emphasize material that has already been covered during the lecture days. The obligation of the instructor is to have a group of class assistants ready to help students with the material. This extra day of class per week is very important because, for the most part, students will show up for the extra day of class to actively participate compared to an out-of-class session. Moreover, students are less likely to visit the instructor in his office during office hours, nor go to a mathematical tutoring center. For the instructor, SP days is the most efficient way to help many students at the same time. There would be no way for the instructor to help this many students during office visits and reduces the task of explaining material many different times to different students during office hours. The worked example worksheets act like a tutor by presenting students with a number of examples and problems to practice. Since it was shown that the high and low prior mathematical knowledge group did not benefit, other modes might be designed to help these students. For example, maybe smaller group session using the worked example worksheets with focus on helping the students would benefit the low prior mathematical knowledge group more.

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