WHAT DO LECTURE TEACHERS BRING TO A STUDENT-CENTERED CLASSROOM? A CATALOGUE OF LECTURE TEACHER MOVES

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In this article we aim to understand what it looks like when community college instructors, with little to no experience with inquiry-oriented curriculum, implement inquiry-based curriculum for the first time. To approach this question we focused our attention on a community college instructor's first implementation of an inquiry-oriented task. In total we identified six moves used during the implementation of this task. These moves are (1) zooming out, (2) real world examples, (3) counter-examples, (4) selective restating (5) referring to definitions, and (6) sequencing of student sharing. By identifying these teacher moves, it is indicated that mathematics instructors, even ones who primarily engage in teacher-center teaching, have techniques that they can draw on as they enact inquiry-oriented curriculum materials. Identifying such techniques can serve as a starting point for understanding how to support college-level teachers in changing their teaching practices.

Key Words: Inquiry-oriented, Teaching, Teacher Moves, Community College

Introduction

At the turn of the millennium, mathematics education reformers placed a renewed importance on inquiry-based instruction. The National Council of Teachers of Mathematics (2000) recommended that "teachers [should] help students make, refine, and explore conjectures," and that students should become "flexible and resourceful problem solvers". However, the pedagogical skills necessary for implementing inquiry-based tasks may differ from those of lecture based pedagogy. Although the needs of reform based instruction have been explored at the K-12 grades (Ball, 1993; Bowler, 1998, 2006; Cohen, 1990; Wood, 2001) and to a lesser degree at an undergraduate level (Rasmussen & Marrongelle, 2006; Speer & Wagner, 2009), very little research has been done on community college instruction of inquiry-based curriculum. Community colleges are unique environment, both because of the high numbers of non-traditional students and because of the large variability in teacher training and background. Questions about community college instruction become more pressing as enrollment continues to increase (Phillippe & Sullivan, 2006).

We sought to understand what it looks like when community college instructors, with little to no experience with inquiry-oriented curriculum, implement inquiry-based curriculum for the first time. To approach this question we focused our attention on a community college instructor's, Bill's[5], first implementation of an inquiry-oriented task. While Bill had been teaching one and two hundred level mathematics courses, such as pre-calculus and linear algebra, for four years at the community college (and prior to this taught at a high-school for twelve years), this course was Bill's first student-centered teaching experience.

[5] All names in this report are pseudonyms.

Study Context

Our study takes place within the broader context of a project aimed to develop a community college "transition to proof" course, based on an inquiry-oriented abstract algebra curriculum – *Teaching Abstract Algebra for Undersatnding* (Larsen, 2009; Larsen, et al., 2009; Larsen et al., 2011). The task we focus on was the very first mathematical investigation of the course. In this task students were initially given six shapes (see figure 1 below) and were asked to arrange the figures from least to most symmetric. The students worked on this task individually and then in small groups prior to a whole class discussion. The groups shared how they ordered the figures and how they came to that decision. The students were then asked to determine a way to quantify the symmetry of each figure and, using their quantification criteria, the groups ranked the figures and presented both their criteria and their ranking to the whole class. Following these presentations the groups worked to develop both a definition of what a symmetry is and what makes two symmetries equivalent (see Larsen & Bartlo, 2009), the learning goals of this task. In total, this task covered three classes, each one hour and fifteen minutes in duration. The majority of class time was spent with students working in small groups or presenting their ideas in a whole class setting; Bill's role in these classes was to launch the tasks, monitor group work, and facilitate whole class discussion.

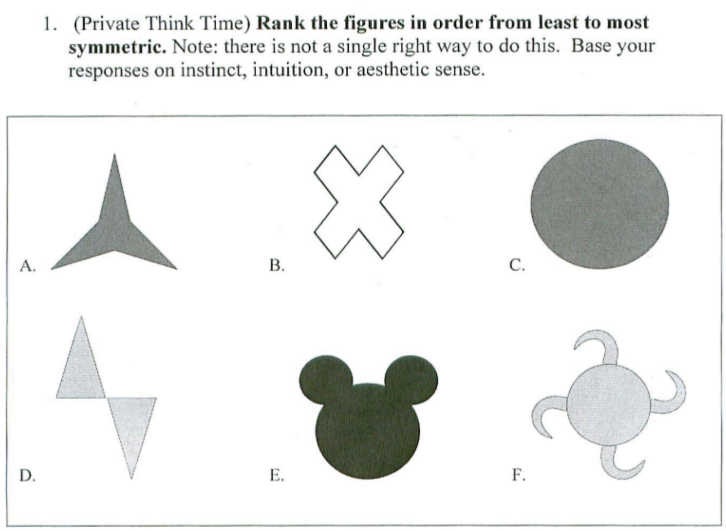

Fig. 1 Symmetry Task Launch

Research Question

Through initial analysis of the classroom video, we refined our research question to focus on the teacher moves Bill had at his disposal during this task. Here we use the term "teacher moves" to refer to specific actions that direct the mathematical trajectory of the lesson, such as providing counter-examples and sequencing student contributions. Specifically, we will examine:

> What types of teacher moves did Bill utilize to direct the trajectory of the lesson as he implemented an inquiry-oriented curriculum task for the first time?

In focusing narrowly on these moves, we are not suggesting that teacher moves exemplify the complexity of teaching practice, nor do we assert that gaining proficiency with specific moves is sufficient for effective teaching. However, understanding what moves teachers may naturally gain though teacher-centered, lecture classes can help inform why teachers struggle to implement reform curriculum and suggest starting-points for professional development and teacher support.

Related Literature

Given that Bill's primary classroom role during this task implementation involved leading and facilitating whole class discussions, we were curious about possible teacher moves that have been shown to advance the mathematical agenda during whole class discussion and student sharing. Other researchers have previously examined the ways teachers negotiate whole class discourse to promote student understanding. Rasmussen and Marrongelle (2006) have proposed a theoretical construct called *pedagogical content tools,* which they define as "a device, such as a graph, diagram, equation, or verbal statement, that a teacher intentionally uses to connect to student thinking while moving the mathematical agenda forward" (pg. 389). They describe and illustrate two such tools, *generative alternatives* and *transformational records* within the context of an undergraduate differential equations classroom.

Additionally, Speer and Wagner (2009) investigated some of the challenges faced by Gage, an experienced mathematics professor, in his first attempt to enact an inquiry-oriented curriculum in a differential equations course. The authors focused on *analytic scaffolding*, the teacher contributions that support the development of mathematical ideas for students. They found that deficiencies in Gage's pedagogical content knowledge led to an inability to create sufficient analytic scaffolding to support classroom discourse and to move the mathematics forward.

We approached our data set with an eye tuned toward teacher tools such as pedagogical content tools and analytic scaffolding, however we were also interested in identifying other teacher moves that both moved the classroom towards the intended goal of the task and contributed to student mathematical sense-making, such as questioning and sequencing. We use the term teacher moves, as opposed to tools, to reflect our focus on any specific teacher action that redirects the trajectory of lesson as it is unfolding in the moment. This contrasts with the formal notion of tools, which were defined by Rasmussen and Marrongelle (2006) as "something that the informed user explicitly recognizes as useful for achieving specific goals" (p. 389). Therefore, when looking for teacher moves we did not require that Bill explicitly recognized their usefulness or that these moves were done with a conscious intention.

Methods

All of the three class periods in which the symmetry task was implemented were video recorded, with the camera focusing on Bill during whole class discussion and on a single group of students during individual and group work. The research team made several passes through this video data, consistent with iterative video analysis (Lesh & Lehrer, 2000), in order to refine our initial research questions and then to identify teacher moves used by Bill to facilitate whole class discussions and facilitate student construction of understanding.

During the first pass of analysis, a video log was made, in which time stamps were recorded to identify the type of class activity (i.e. group work, poster presentations) and to make note of times in which Bill was leading/facilitating whole class discussion. Additionally, key mathematical moments of the task were identified, such as when the definition of symmetry was introduced to the class. It was during this phase of analysis that the research team began to recognize specific instances in which Bill stepped in to direct the trajectory of the lesson. For instance, when the students were initially given the figures and were asked to order them from least to most symmetric, some students were basing their decisions on how many times you could fold the figure such that the two halves would be the same. Upon hearing such reasoning during small group discussions, Bill redirected students to think of reflecting the figures as opposed to folding the figures[6]. In describing the mechanism in which Bill redirected the class away from folding, the research team became curious as to other moves used by Bill during the implementation of this task.

In the second phase of analysis, each member of the research team watched one or two of the three days of classroom video data, this time with the specific goal of identifying moves used by Bill to direct and facilitate whole class discussion. The research team then met to share possible moves and the video clips in which these possible moves were present. During this final stage of analysis the research team reached a consensus about both which moves were present and which clips were illustrative of these moves.

Results and Discussion

In total we identified six moves used by Bill over three days, during the implementation of this task. These moves are (1) zooming out, (2) real world examples, (3) counter-examples, (4) selective restating (5) referring to definitions, and (6) sequencing of student sharing. In this section we will describe each of these moves and provide examples of how Bill used these moves during whole class discussions.

Zooming Out

Twice during the implementation of this task, Bill made reference to "how things will be done in this class" or to "how the mathematical community works" as a way to motivate or justify a change in the trajectory in the lesson. One example of this was when Bill introduced the definition of a symmetry to the class. At this point in the task, each group had just presented their ranking of the figures and the criteria they used to quantify the symmetry of a figure. In an effort to motivate why Bill would give the class a definition of symmetry, Bill said the following to the class:

> Bill: You are going to be presented *in this course* with different problems or different puzzles. And often after a little playing around everybody is going to end up on pretty much the same track, because there is just not so many ways to approach them. Other times there really is a lot of different ways to approach it, and so there is no kind of structure that would kind of, put everyone in the same channel. When that happens, which will happen every now and then, *that's when I'm going to step in and present some formal definition, or something like*

[6] Bill was guiding the class towards a definition of symmetry as a type of rigid motion. Identifying symmetries in terms of "folds" is in conflict with the idea of rigid motion.

> *that*. *That will happen occasionally as we go through the course*. And if I were a student in your shoes, I would feel comforted with that, that I'm not inventing mathematics that is different from what everyone else is doing.

In this statement, Bill described a classroom norm: in times of classroom debate, Bill may step in to provide formal mathematical definitions. In this specific instance, Bill's use of referring to this norm served to justify the rapid change in trajectory of the class. Directly following this explanation Bill presented a definition of symmetry, stating that "a symmetry is a rigid motion, that when applied to a figure, results in the figure landing on itself". This definition of symmetry had been used by previous instructors of this curriculum. So, this definition was developed prior to Bill's implementation of this task and thus in absence of his students' emerging conceptions of symmetry. By introducing the definition of a symmetry to the class this way, as opposed to building off of the ranking criteria developed by the students, such as "the number of ways to reflect the figure to make it land on itself", Bill drew the attention of the class away from the students' work on the task and towards a more general description of how this course will operate. Thus, this serves as an example of how Bill *zoomed out* as a way to direct the trajectory of the lesson.

Real World Examples

Three times in the implementation of this task, Bill made use of real world examples to clearly illustrate mathematical ideas. In both of the following excerpts, the examples served as analogies for mathematical definitions. In the following transcript, Bill builds upon the student description of what it means for a figure to "land on itself." The student described "landing on itself" to mean that all of the boarders aligned on one another. Bill elaborated further with the following:

> Bill: I'm wondering, what's a good image that I could use that a lot of us are going to really connect with well. And the mentioning of boundaries made me think of, um, toys that toddlers and sub-toddlers play with. You know those toys that help toddlers figure out what's a circle and what's a square, and what's a star. And so it would be like some board, with a kind of die cut shapes, and try to fit the shapes in. That sounds like what you are describing.
>
> S1: Yeah.
>
> Bill: Does that give everyone kind of a common framework. I wanted to give some sort of vivid image to support what we have been doing.

In this passage it is clear that Bill wished to establish a common classroom conception of "landing on itself" by tying the idea to a real world child's game that he hoped was common to the students. This analogy was useful to at least one student. Later in the class period when students were discussing equivalence, one student made reference to "fitting the star into the star hole," which built upon Bill's initial imagery.

In the next example, Bill settled a class dispute about the distinction between the ideas of "equivalence" and being "the same." The following transcript begins with a student explaining the similarity between a 0 angle and a 2π angle.

S1: Is it [a zero angle] the same as a 2π angle?
Bill: Is it?
S2: How can you tell the difference?
S1: They have the same sine and the same cosine.
S3: But, he's asking if equivalent means the same.
S4: You aren't doing the same, but you end up with exactly the same.
S3: So it probably wouldn't be, because you are doing two different symmetries that are equivalent, but not the same.
Bill: *So if you performed a 360° rotation on yourself, versus a 720 degree rotation on yourself, it is equivalent, but is it identical?*
Students: No, no.
S1: The more you do the dizzier you get.
Bill: So is a 360° equivalent to a zero degree?
S5: No.
Bill: Equivalent?
S5: Oh, equivalent yes.
[other students say yes]
Bill: Identical?
Students: No.

In this passage, Bill used the real world example of a student physically spinning in circles to draw a clear distinction between "the same" and "equivalent." Through the use of the example, students were led to the mathematically conventional conception that equivalence is not the same as being identical, which is a crucial concept in abstract algebra.

It is important to note that this type of teacher move, introducing illustrative examples from a real world context, may be a valuable technique within a lecture-based classroom. In teacher-directed classes, teachers are wholly responsible for presenting material in a clear manner that is easily assimilated by the students. Real world examples can help students visualize or understand abstract material presented by the instructor. During the inquiry-based task, Bill carried over this technique to "settle" student disputes and to restate a student idea in a more vivid way.

Counter-Examples

Another technique that Bill used to advance student understanding was to provide counter-examples to challenge student conceptions. His use of counter examples was similar to the idea of generative alternatives proposed by Rasmussen and Marrongelle (2006). However, the primary difference between Bill's use of counter-examples and generative alternatives is that generative alternatives typically serve to establish norms or promote student justification, whereas Bill typically used counter-examples to highlight deficiencies in student thinking. In addition, generative alternatives are a written record of some form, whereas Bill's counter-examples were purely visual or verbal.

In the following excerpt, Bill was leading a whole class discussion as the class worked to define "rigid motion", a concept that was crucial to the class's definition of symmetry[7].

> Bill: If a rectangle is my figure, is that a rigid motion? [Bill rotates the rectangular eraser 90°.]
> Class: Yes.
> Bill: And what makes that a rigid motion?
> S1: It has a pivot point.
> Bill: It has a pivot point. What else?
> Ellie: It's not changing the object itself.
> Bill: It's not changing the object itself. Is this a rigid motion? [Bill moves the eraser 12 inches up and to the left.]
> Class: Yes.
> Bill: Does it have a pivot point?
> Class: No.
> Bill: Ok, so a lot of the rigid motions we have been dealing with have pivot points, it doesn't seem to be a requirement of a rigid motion.

In this interchange, Bill did not accept the definition provided by student 1. Instead, Bill provided a counter-example that did not fit student 1's definition of a rigid motion, but did fit the class's intuition about what a rigid motion should be. By using a counter-example, Bill was able to clearly illustrate to the class that first student's definition of rigid motion was inadequate, and he directed the class toward the desired definition.

Selective Restating

In addition to providing counter-examples, another move that Bill used to guide classroom discourse down productive pathways was selective restating. Within discourse analysis, the term revoicing has been defined as "the reuttering of another person's speech through repetition, expansion, rephrasing and reporting" (Forman, McComrick & Donato, 1998, p. 531). O'Connor and Michaels (1993) claim that teacher revoicing is a useful teaching tool with many purposes, including allowing students to claim (or disclaim) ownership of ideas, giving credit to student ideas while allowing teacher influence to create "warranted inferences," and lending authority to hesitant voices. Our use of the term restating includes both this definition of revoicing as well as asking students to restate their own speech. Asking students to restate previous ideas does not allow the teacher to directly expand on the student idea, however it has many of the other benefits of revoicing, such as lending authority and ownership to student ideas while influencing the focus of classroom discourse. In addition, restating can promote student empowerment through direct contribution, and it allows students the opportunity to practice reformulating their contributions for the class.

The following transcript except took place during whole class discussion of the definition of rigid motion, just after the dialogue illustrating Bill's use of counter-examples presented previously.

> Bill: And so, Ellie, what again, was your way of expressing that, what a rigid motion is? I did this [moves eraser 12 inches up and to the left], and you had a great explanation just now.

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[7] Earlier in this class Bill had defined a symmetry as a rigid motion, that when applied to a figure, results in the figure landing on itself.

Ellie: Something that does not affect the object itself, from changing.

This conversation provides an example of asking a student to restate a previous idea. Bill selected a student contribution that he recognized as mathematically valuable, and he redirected the class's attention to that contribution. In this case, Bill did not question Ellie's definition further or add additional commentary. He accepted this idea and moves on to another topic.

Appealing to Definitions

Yet another pedagogical move that Bill made use of in this task was appealing to definitions. Previous to the conversation in the following transcript, the class had spent almost half of the class period reviewing the definition of symmetry. In this exchange, the students were asked to think about whether a 360° rotation was a symmetry or not. In the following passage, Bill engages the class in conversation.

Bill: Let's see how people are landing on this. Um, how many people say a 360° rotation is a symmetry. [Many Students raise hands] How many people say that it is not? [Only one student raises a hand.] Let's hear from the not. Why isn't it a symmetry?

S1: Uh, I just think it is kind of arbitrary. Because, like, if, if you want to be able to, like, count it, how are you supposed to count something if you can just, like, make it go as many times as you want it to.

Bill: Well, the question we are asking is, does it meet the definition of symmetry? Is a 360° rotation a rigid motion?

S1: Yeah.

Bill: When you apply it to the figure, does it result in the figure landing on itself.

S1: Yeah.

Bill: So is it a symmetry?

S1: Yeah.

The dissenting student in this passage partially articulated one of the dilemmas associated with equivalence. He stated that, "It is kind of arbitrary… [because you can] make it go as many times as you want it to." The difficulty that the student had might be solved by introducing the idea of equivalence classes; each of the rotations that are multiples of 360° fall into the same equivalence class and are thus not thought of as contributing to the total number of symmetries. Instead of building upon the mathematical insights of this student and opening up a discussion about equivalence, one of the goals of this task, Bill instead chose to appeal directly to the classroom's accepted definition of symmetry to convince the student that a 360° rotation is a symmetry. Bill's appeal to definitions directed the mathematical trajectory of the class away from the mathematically significant conflict experienced by one student and toward the classroom's accepted definition of symmetry.

Sequencing of Student Sharing

Organizing student contributions into logical order is one technique that teachers use to guide the flow of classroom discourse. Bill evidenced intentionality in the sequencing of the group presentations. In one instance during the small group time, Bill visited each of the groups and evaluated the maturity of each group's definition. He then selected the group that had the least complete definition to share first.

> Bill: This might be a good time just to share, kind of, where we are in the process for each group. And I want to go ahead and start with you guys because *I know that you are kind of still in the middle- more in the process of being formed*, but you have some of the idea. Can one or two or all 4 of you kind of articulate where you are with this?

The final group that Bill called on was the group that had the most complete and most formal definition. Bill had this group write their definition on the board and used it as a launching point for the next discussion.

Although Bill did make use of intentional sequencing to aid in the trajectory of lesson development, Bill's sequencing strategy is somewhat incomplete. Bill mentioned that he did not know the status of the second group's definition. Thus, although Bill indicated an awareness of intentional sequencing of some of the group presentations, his implementation did not take into account the state of one of the groups. Regardless, we see that in this case Bill was able to guide the trajectory of the lesson toward his intended goal even with this incomplete knowledge.

Conclusions

This work serves as a preliminary investigation into Bill's existent teaching techniques in his first implementation of an inquiry-oriented task. We found that he utilized a number of moves, including zooming out, illustrative real world examples, counter-examples, selective restating, appealing to definitions, and intentional sequencing of student sharing. Understanding the moves Bill utilized helps to paint a picture of some of the challenges he faced when opening his class to student-focused instruction and forms a basis upon which further professional development can occur. By identifying these teacher moves, it is indicated that mathematics instructors, even ones who primarily engage in teacher-center teaching, have techniques that they can draw on as they enact inquiry-oriented curriculum materials. Identifying such techniques can serve as a starting point for understanding how to support college-level teachers in changing their teaching practices. For instance, given the moves we identified, we can assume that Bill saw benefit in restating student contributions and utilizing examples (both counter and vivid). Therefore, motivating these tools may not need to be a focus of professional development. Instead, professional development could be designed to support Bill to shift the responsibility of restating and introducing examples to the students.

This study is just a starting point in an investigation of pedagogical techniques for student-centered teaching at the community college level. From this information, other important questions emerge, such as whether Bill will continue to use these moves through the semester, or whether the moves are modified or augmented through his experience with this curriculum. It would also be interesting to track Bill's pedagogy in courses outside of this class in order to study the influence of this inventive curriculum on Bill's pedagogy. Another possible goal for the extension of this research would the construction of a hypothetical learning trajectory (Freudenthal, 1991) for how lecture-based teachers learn how to implement inquiry-based curriculum. A more general hypothetical learning trajectory for teachers could be informative for professional development programs that support teachers in transition to student-oriented instruction.

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