Sociomathematical Norms: Under Whose Authority

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The study of sociomathematical norms initiated by Yackel and Cobb (1996) has become a popular way to make sense of the complexity of mathematical activity in the classroom. In this study we explore the role authority plays in the negotiation and legitimization of sociomathematical norms. We found that sociomathematical norms in this setting were introduced with hierarchal authorities whether the expectation was introduced implicitly or explicitly. We also found, when sociomathematical norms were introduced by a student, mathematical authority played a strong role in the negotiation process including interpretation, and legitimization.

Key words: sociomathematical norms, authority, inquiry-based, calculus, undergraduate

Introduction

In 1996, Yackel and Cobb introduced the study of sociomathematical norms in an attempt to understand how students' mathematical autonomy might be fostered by their mathematical beliefs and values and to make sense of the complexity of mathematical activity in the classroom. They defined sociomathematical norms to be "normative aspects of mathematics discussion specific to students' mathematical activity" (p. 461). In other words sociomathematical norms can be seen as the reoccurring mathematical aspects of discourse that focus on mathematical thinking rather than thinking about mathematics. For example, the social norm that you should justify your answer does not, by itself, insure that your justifications will be accurate, rigorous, or convincing. However, a sociomathematical norm that defines what constitutes a convincing argument can be introduced to set an expectation in the classroom that encourages strong mathematical activity in the form of justification (Yackel & Cobb, 1996; Kazemi & Stipek, 2001).

In 2005, Levenson, Tirosh, and Tsamir found that in a traditional setting with the teacher as authority over the introduction of sociomathematical norms, the teacher and students did not share the same interpretations of a sociomathematical norm. So while, an expectation was introduced, it did not become normative. We suspect that there was a breakdown in the negotiation processs wherein students and the teacher interpret and agree upon the sociomathematical norm. In this study we address the problem identified by Levenson et al. by exploring the role authority plays in the negotiation and legitimization of teacher- and student-initiated sociomathematical norms.

Literature Review

We first discuss research on sociomathematical norms. We distill the literature into a framework that we use to identify and study sociomathematical norms. Finally we share the results of a study in which we analyzed the same data presented here to gain insights into students and teachers use of authority in this inquiry-based setting.

Sociomathematical Norms

Research on sociomathematical norms has covered a sampling of educational settings with most focusing on elementary school mathematics classrooms. Sociomathematical norms have been documented by researchers in first grade (McClain & Cobb, 2001), second grade (Yackel and Cobb, 1996) third grade (Mottier Lopez & Allal, 2007), fourth and fifth grade (Kazemi & Stipek, 2001, Levenson, Tirosh, and Tsamir), junior high (Hershkowitz & Schwarz, 1999), and university (Yackel, Rasmussen, & King, 2000). Some of these classes have been enriched by technology, some have been taught by teachers attempting reformed-minded practices for the first time, and some have been taught by researchers themselves. In all cases, the studies reflect the admonition put forth originally by Yackel and Cobb (1996), "[they] limit [their] discussion to classrooms that follow an inquiry tradition. Nevertheless, sociomathematical norms … are established in all classrooms regardless of instructional tradition" (p 462).

Sociomathematical norms are reported as general categories among which there are multiple ways of defining a norm. For instance, the negotiation of what constitutes an acceptable mathematical explanation (Levenson, Tirosh, and Tsamir, 2005, 2009 ; Mottier Lopez and Allal, 2007; Yackel and Cobb, 1996) and what constitutes a mathematical difference (Mottier Lopez and Allal, 2007; Yackel and Cobb, 1996). These categories are not themselves sociomathematical norms, but represent some of the types of sociomathematical norms that might be present. For each category, there is a continuum of different sociomathematical norms that could be enacted to structure the mathematical activity in the classroom. For example, an acceptable mathematical explanation in one classroom may require only a description of the procedure done to achieve the answer and in another classroom may require a mathematical argument. The mathematical activity expected in crafting and presenting an explanation and in listening to an explanation in these two classrooms would be quite different even though both had established sociomathematical norms that guided the negotiation of what constitutes an acceptable mathematical explanation. The students in the classroom with the stronger sociomathematical norm for explanation will be required to engage in a higher level of mathematical activity.

Originally Yackel and Cobb (1996) focused on the introduction of sociomathematical norms by the teacher. Since then, researchers have documented sociomathematical norms introduced by a research team (McClain & Cobb, 2001), teachers (Yackel, Rasmussen, King 2000), or students (Hershkowitz & Schwarz, 1999). Once introduced, sociomathematical norms are negotiated and re-negotiated by various participants in the class. This ongoing negotiation of sociomathematical norms can be initiated by either student or teacher through interactions: teacher-to-student, student-to-student, or students-to-tool (Yackel & Cobb, 1996, Hershkowitz & Schwarz, 1999). After the expectation is communicated and understood, "whenever someone acts in accordance with [that] expectation, they contribute to the ongoing constitution of the expectation as normative in that situation" (Yackel, Rasmussen, King 2000, p 281).

If the sociomathematical norm is pre-planned and introduced by the teacher (as in: Yackel and Cobb, 1996; Levenson, Tirosh, and Tsamir, 2005) there is little need for a framework that could be used to identify a sociomathematical norm within the social interaction of a class discussion. However, in order to study a sociomathematical norm in its developmental stages, in the classroom, Staub (2007) suggests that one must further focus on the interactions between the teachers and the students as did Mottier Lopez and Allal (2007). At this point we address a concern put forth by Mottier Lopez and Allal (2007) that Yackel and Cobb's (1996) "subtle distinction [between social and sociomathematical norms] does not provide a sufficiently clear foundation for empirically grounded interpretations" (2007, pg 254). While our data and questions are similar to those of Mottier Lopez and Allal (2007) Our solution is not to minimize the difference between social and sociomathematical norms because we agree with Kazemi and Stipek (2001) that "the distinction between social and sociomathematical norms is useful for studying how classroom practices move beyond superficial features of reform" (2001, pg 60). To maintain the strength behind sociomathematical norms and to further articulate the distinction between social norms and sociomathematical norms we have systemized the definition of sociomathematical norms thereby creating a framework through which sociomathematical norms can be identified in videodata of an inquiry-based mathematics class.

As noted above, it is generally agreed that three components must be identified for an expectation to qualify as a sociomathematical norm. Though there are three distinct components, we are not suggesting a hierarchal or chronological progression. The three components of a sociomathematical norm are: 1. a mathematical expectation is set forth (Yackel and Cobb, 1996) 2. the expectation is negotiated by the participants (Yackel & Cobb, 1996, Hershkowitz & Schwarz, 1999), and 3. the expectation becomes normative (Yackel & Cobb, 1996; Hershkowitz & Schwarz, 1999). The negotiation process in component 2 can further be broken down into three sub-components. Within the negotiation process a) a mathematical interpretation of the expectation occurs (Yackel & Cobb, 1996), b) the expectation is agreed upon (Yackel & Cobb, 1996; Yackel, Rasmussen, and King, 2000), and c) the expectation is validated as legitimate. This last subcomponent of negotiation of a sociomathematical norm was never directly discussed in the research but was alluded to by Levenson, Tirosh, and Tsamir (2005), when they suggested that one student, Dan, preferred mathematically based explanations throughout the course even as the teacher explicitly presented the sociomathematical norm that practically-based explanations were preferable. In this case, Dan rejected the teacher's expectation because he did not see the practically-based explanations as legitimate. Therefore, we believe that the validation of the expectation is an important part of the negotiation process that is essential for an expectation to be agreed upon and to become normative.

Since, in this paper, we are mainly concerned with the introduction and negotiation of sociomathematical norms, we feel it necessary to define two new terms. We will refer to a sociomathematical event[4] as an event where a component of a sociomathematical norm is present. So for instance if an expecation is being introduced, that would be called a sociomathematical event. We will define a quasi sociomathematical norm as a group of sociomathematical events that have some but not all of the components of a sociomathematical norm. For example, if an expectation has been introduced and the participants in the class are in the process of negotiating the expectation, but the expectation has not yet become normative, we would call it a quasi sociomathematical norm. This allows us to focus on the developmental stages of a sociomathematical norm without following it to the point at which it becomes normative.

Authority

In previous work, we suggested authority in the classroom hinges on three major concepts: authority relation, legitimacy, and change (Gerson & Bateman, 2010). The authority relation is a

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[4] We are using sociomathematical event in essentially the same way as Levenson, Tirosh, and Tsamir (2005) use the term meta-discursive rules. However, we feel that the term sociomathematical event better captures the connection between the components and the sociomathematical norm itself.

relationship between two or more people, with at least one person acting as the bearer of authority and at least one person acting as the receiver of authority. The bearer makes a claim; the receiver recognizes the claim as legitimate and is influenced to change his or her behavior or thinking. In traditional classroom settings, authority is usually hierarchal with the teacher acting as bearer of authority and the student acting as receiver of authority (Herbel-Eisenmann, Wagner, & Cortes, 2008). In inquiry-based classroom settings the bearer and receiver are more fluid roles taken on by both teacher and student at different times (Hamm & Perry, 2002). We define legitimacy of authority as “the knowledge, skills, position, or experiences that influence a person or group within an authority relationship” (Gerson & Bateman, 2010).

In the same work, we identified four different types of authority held by students and instructors

1) hierarchal authority where legitimization is based upon a person’s position in the class,
2) mathematical authority where legitimization is based in mathematics,
3) expertise authority where legitimization is based upon the expertise of the bearer, and
4) performative authority where legitimization is based upon the bearer’s ability to engage the class. (p. 200)

Of these four types of authority, hierarchal and mathematical authorities both played a major role in the introduction and negotiation of sociomathematical norms as we will further discuss in the results section of this paper. Therefore, we further explain these authorities below. For more information on expertise and perfomative authorities, we refer the reader to our previous paper (Gerson & Bateman, 2010).

Hierarchal authority can be held by students or teachers but has at its source the institutional position of the instructor as bestowed by the institution. A teacher automatically holds this authority and may grant hierarchal authority to a student by inviting her to present to the class. Hierarchal authority is legitimized by the position as instructor or presenter.

We identified two types of hierarchal authority which we called *institutional authority* and *granted authority*. Institutional authority is held by the instructor whose authority is legitimized by her position as instructor of the course and is granted by the institution which hired and assigned her to teach the course. This is a hierarchal authority because the instructor always holds this authority and the students are always subject to it. However, students can also hold hierarchal authority if it is granted to them by someone already holding hierarchal authority. We called this granted authority. For example, when an instructor calls a student to the board to present his solution, the student is temporarily given hierarchal authority over the other students in the class. He has the authority to direct the class discussion and to call on other students to ask or answer questions. Hierarchal authority has nothing to do with the mathematics that is being presented or discussed, but is legitimized by the student’s position as presenter which was granted to him by the instructor.

Mathematical authority is invoked whenever a student or teacher explains or justifies a mathematical statement or uses previously explained mathematics to legitimize a claim. The mathematical authority relation is sealed when a student accepts the mathematical argument and changes his thinking or actions to accommodate the argument.

In addition to the two types of hierarchal authority, we identified two types of mathematical authority, *justification authority* and *mathematics community authority*. In both of these cases a mathematical argument legitimizes the claim. When a person holds justification authority, the bearer presents a mathematical argument and the receiver recognizes the legitimacy of the claim through the mathematical argument. In mathematics community authority the bearer invokes previously argued and accepted mathematics to support his claim. The difference between these two authorities is essentially when the mathematical argument is presented.

One key finding of our study on authority was that the receiver is the one who determines if the bearer holds authority, even in hierarchal authority relations (Gerson & Bateman, 2010). The authority relation is sealed when the receiver recognizes the legitimacy of the bearer's claim and is influenced to change. Therefore, even if the bearer has no intention to bear authority, the receiver can bestow authority upon the bearer. And if the bearer acts with intention to bear authority, but is not legitimized by the receiver, there is no authority relation.

Consequently, the classroom teacher may not be able to limit the power of her own institutional authority. For the teacher, all types of authority are naturally subsumed by the institutional authority that she also holds. It is difficult for the teacher to share authority with her students unless she is somehow able to either minimize the influence of her institutional authority or maximize the influence of mathematical authority.

Setting

Our research is set in a teaching experiment (Steffe and Thompson, 2000) in a university honors calculus class conducted by Janet Walter and Hope Gerson. A guiding principle of the design of this class was that agency is at the heart of learning (Brown, 2005; Kohn, 1998; Rogers, 1969; Walter & Gerson, 2007). Agency is defined as "the requirement, responsibility and freedom to choose based on prior experiences and imagination, with concern not only for one's own understandings of mathematics, but with mindful awareness of the impact one's actions and choices may have on others" (Walter and Gerson, 2007). We therefore designed the calculus class to maximize students' exercise of agency. Students worked on tasks designed or selected to elicit conceptually important calculus content without prior instruction. The teaching experiment consisted of two Calculus I classes taught in the fall 2006 and the winter 2007, followed by a Calculus II class taught in the fall 2007.

The corpus of data from this study is taken from two two-hour class periods in the Calculus II class taught in the fall of 2007. In these class periods, students from the four collaborative groups worked on the Hemisphere Tank Task and presented solutions to the class. These class periods were chosen for two reasons. First they occurred early in the semester, in the fourth week of class, as sociomathematical norms were still being negotiated. Second, a compelling episode occurred at the beginning of the second day of the task, where Michael, a student in the class, introduced a new way of thinking about what constitutes a mathematical difference and introduced a new expectation to the class about how mathematical difference should be explored. We recognized this episode as pertaining to the negotiation of sociomathematical norms and wanted to further understand the dynamics in play pertaining to sociomathematical events and quasi sociomathematical norms in this episode. In order to understand this single episode, we felt it necessary to analyze the surrounding mathematical activity with the following questions in mind: 1) what quasi sociomathematical norms are enacted in the two class periods? and 2) under what authority are expecatations introduced? 3) What roles does authority play in quasi sociomathematical norms?

The Hemisphere Water Tank Task

The Hemisphere Water Tank Task (Figure 1) is a common homework problem in Calculus II. It usually appears in a text after the development of the disk and/or shell methods. We used this task quite differently. We gave the task to students before they had learned the disk or shell methods. The completion of the task required students to develop a method for calculating the volume of a solid of revolution and justify their methods and answers.

Hemisphere Water Tank

A certain water tank is a hemispherical bowl of radius 5 feet. Water flows into the tank at the rate of
10 cubic feet per minute.

A. Find the volume of the water in the tank when the water is 3 feet high at the center of the tank.

B. Find an equation for the volume of the water in the tank for any given height, *h*, of the water at the center of the tank.

Extension: How fast is the water level rising when the water is 3 feet high at the center of the tank?

Figure 1: The Hemisphere Water Tank Task

Method

We analyzed four hours of videotape gathered on January 29, 2007 and January 31, 2007. All episodes were transcribed and independently verified by the research team and we proceeded to code the videodata for key ideas, such as authority, agency, social norms, and sociomathematical norms. Together we coded parts of the compelling episode mentioned above. Then we independently coded surrounding episodes. We built consensus about how to define each code and the importance of each code. After identifying sociomathematical events, the first author viewed the 3 weeks of videodata prior to the corpus of data to identify the first occurrence of each quasi sociomathematical norm.

We then used axial coding using the report and key word map features of Transana (Woods and Fassnacht, 2007). During this process we looked for connections among the different key ideas. When connections seemingly emerged from the axial coding, we went back to the original video and transcripts to validate these connections.

Results

We identified sociomathematical events for three quasi sociomathematical norms and one sociomathematical norm during the presentations that defined *what constitutes a good explanation* and *what constitutes a mathematical difference*. In particular, 1) a good explanation consists of connecting the model, context, and equation, 2) a good explanation provides a mathematical argument 2) a different model creates different mathematics, and 3) two equations are different if they act differently on a common model.

What constitutes a good explanation?

In the first presentation, Group One divided the sphere into "a bunch of little cylinders." They found an equation for the volume of each cylinder, $\square=\square\square 2h$, and created a limit of a sum that would calculate the volume of all of the cylinders $\lim \Delta h \rightarrow 0 1 h \Delta h \square\square 2 \Delta h$. In his explanation, Paul invoked the quasi sociomathematical norm that a good explanation connects the model, context, and equation. Paul explained his groups' sum, by connecting the model, "the cylinders" in lines 1 and 5, the context "the height of the water" in lines 1 and 5 and the equation

"□□*2*times the height" in line 1. Paul consistently demonstrated that he defined a good explanation as including connections among the model, context and equation.

1 Paul: So to get it so this equation would work, we have to get the height of the cylinders going to zero. It's going from the first cylinder to the height of the water over Δh, where—which would give us the amount of cylinders we have. And times the, uh or no, summing up each little cylinder, just □□*2* times the height of each cylinder, and the limit of a sum. Do you guys get it so far? Questions? [Timbre raises her hand]

2 Derrick: Yeah.

3 Paul: 'Kay, yeah.

4 Timbre: Can you go over the, the h divided by Δh? What was that for again?

5 Paul: 'Kay, so the height of the water is right here—is how high each little cylinder is. So if you divide the height by how high each cylinder is, that will give you how many cylinders we have. So you go from the first cylinder to the last cylinder. Do you get that?

6 Timbre: Yeah.

In our next example, the second group was presenting and John asked them to affirm his own interpretations of the variable □*0*. Michael, in line 7, offered an explanation that connected the model and the equation, but did not connect the context. The instructor, Janet, in line 9, implicitly reintroduced the expectation that a strong explanation includes connections among the model, context, and equation.

7 Michael: [□*0*] is the radius of like the cylinders that we are gonna be making here in a minute.

8 John: Okay.

9 Janet: So I also think of that new r as if I am looking at the surface of the water at a particular height? It's the radius of that [Michael: of that circle that it makes] circle that the—of the surface of the water.

10 Michael: Right, you take the bowl and you look at it from above, that's the radius of—of the water level at that point.

After hearing Michael's explanation John's response of "Okay" and his subsequent silence indicate that John deemed Michael's explanation as sufficient (or was unable to provide another response that would improve Michael's explanation). Even after the clarification was given and apparently accepted, Janet implicitly suggested a further expectation to connect the explanation of the meaning of □*0* not only with the model of the problem, but also with the context in line 9.

When Paul presented the explanation of his group's equation, lines 1 and 5, he invoked the quasi sociomathematical norm that a good explanation connects the model, context and equation. Later, when Michael presented, his group's equation, he failed to present an acceptable explanation. Janet extended his explanation to include the context, further negotiating and legitimizing (with her institutional authority) the quasi sociomathematical norm of what constitutes a good explanation. This shows that at this point in the class, while there was some agreement on what constitutes a good explanation, the quasi sociomathematical norm had not yet been completely agreed upon and adopted by all of the students in the class. It was still in the developmental stages.

On the other hand, when class members were explaining new mathematical procedures or models, they nearly always included mathematical arguments to support those procedures. For instance, when Paul initially introduced the idea of adding up the volume of a bunch of little cylinders he explained why the limit of the sum, as the height of the cylinders goes to zero, would be necessary.

11 Paul:	‘Cause first of all, in our sphere [makes bowl shape in air] the cylinders [uses thumb and finger to show height] wouldn't be exactly cylinders [tilts hands and pantomines rubbing up and down side of bowl] anyways, because the sides would be angled in. [points to picture of bowl cut in cylinders] So to get it, so this equation would work we have to get the height [uses finger and thumb to show height] of the cylinders going to zero.

In this case, Paul essentially argued that in order for the volume of a cylinder to be an accurate approximation of a cross-sectional slice of a bowl, one would have to take the limit as the height of the cylinder goes to zero.

In the next transcript, in line 12, Tyler explained Group Two’s model for the first time. Since the model was different than the first model presented, the group was very careful to explain how they constructed their equations. Tyler presented a mathematical argument for $\square 2+\square 2=25$. Then later in line 14, Heber supplied a mathematical argument for $\square=h-5$.

12 Tyler:	We found out that when you use uh, Pythgorean coordinates, you know by saying that this length here [traces the diagonal radius] on this line is always going to be 5, cause from the center point of the sphere pointing to all points on the outside of the sphere, the radius of the sphere is 5. It's always going to be 5. But we are going to have x and y and y could be our $5-h$ or even you could go backwards and do $h-5$. But then we would be able to find that and just using you know $\square 2+\square 2=25$. Because you have that, you can find out what x and y are.
13 Michael:	That’s—is everybody clear on the, we just redefined the same variable names that had been used—looks like before, but we defined them as different things. And to understand everything that we are going to do, we need to understand what those represent. So, [Heber: Yeah.] That’s kind of important.
14 Heber:	So is it clear how we got y equal to h minus 5?
15 Daniel:	Um, can you just re-go over that, like write out the steps that you did
16 Heber:	We know that $\square=h-5$ 'cause, uh, like if it was negative 5 the height would be 0. So zero minus five is negative five and that's where it is on the y-axis [pointing to diagram]. Or also like the example here, y would be -2 like they had done on their's. Uh, so if y is -2 the height is 3, $3-5=-2$. So just did this to convert y into h. And so we were able to substitute this $h-5$ value into our equation up here [points to equation].

Because the students were regularly providing a mathematical argument to support new models and procedures, we have evidence that in the first three weeks of class, the expectation that an explanation of a new mathematical procedure must include a supporting mathematical argument, had already become normative. Therefore we can say that this was an instance of a sociomathematical norm for what constitutes an acceptable explanation.

What constitutes a mathematical difference?

At the beginning of the second presentation, Tyler and Heber explained their model. As the other group did, they used *r* to represent the radius of each little cylinder. And in contrast to Group One, who used *h* and *y* interchangeably, group two used *y* to represent the *y*-coordinate and *h* to represent the height of the water. Michael suggested, in line 13, that "we defined them [the variables *x* and *y*] as different things." This invoked the expectation that different models create different mathematics.

After Michael invoked the quasi sociomathematical norm that a different model creates different mathematics, Heber, in line 16, continued to explain his group's connection between *y* and *h*. Despite Daniel's suggestion that he "just…write out the steps that you did" Heber's explanation connected the model, context, and equation, further normalizing the quasi sociomathematical norm of what constitutes a good explanation.

Group One solved the problem using the equation, $-5-2\square(25-h2)\square h$, and group Two solved the problem using the equation, $\square 25-h-52\square h$ .After the two groups presented their solutions, Michael introduced a new expectation (lines 17 and 20) for what constitutes a mathematical difference. Heber and Tyler interpreted the expectation (lines 18 and 19) and Michael began to negotiate the meaning of the new expectation (line 20).

17 Michael: Now, we it looks like we've got two different equations? *{expectation}*
18 Heber: Yeah. *{interpretation}*
19 Tyler: But they're the same. *{interpretation}*
20 Michael: Um, they're not the same equation, they both model something different, and I'm 90 percent sure that I know what that is, the difference. *{expectation and negotiation}*

In order for this new expectation that two equations are different if they "model something different" to become normative, Michael's expectation needed to be interpreted, agreed upon, and legitimized. For the next ten minutes, Michael and the class continued to negotiate the meaning of Michael's expectation of what constitutes a mathematical difference. For example in the next excerpt, Michael re-states the expectation embedded in a mathematical argument in line 23, and Robert interprets that to mean solving part A, and expresses that he understands Michael's expectation.

21 Michael: if you just get the volume function and just start evaluating the volume function *{mathematical argument}*
22 Derrick: [inaudible] you could get a general equation *{interpretation}*
23 Michael: Right, but I'm saying take an indefinite in, er a definite integral of their equation. What would that model? What, if you plug in the value of one, into their indefinite integral [sic], what does that represent? *{mathematical argument, mathematical authority, t and restatement of the expectation}*
24 Robert: 'Cause, 'cause do you want me to solve part A? 'cause it [inaudible] [moves thumb and index finger together] I see what you're asking. *{interpretation}*

Michael's initial statement in line 21, that "they both model something different" did not supply enough mathematical information for Robert and the rest of the class to build a mathematical interpretation of the expectation, nor to judge whether it was legitimate. When Michael began to explain his expectation he made explicit how he wanted to compare the two equations, and offered legitimization of the expectation through mathematical authority. Although Michael held granted authority to present his ideas to the class, and expertise authority for his own solution, it was Michael's mathematical authority that legitimized his expectation through his mathematical argument, and allowed others to begin to interpret and agree on the expectation.

In the next excerpt, Michael re-stated the expectation in line 25, and Michael and Heber provided an explanation that included connections among the model, context, and equation as well as a mathematical argument for how they were interpreting the two equations. They explained that they used their own procedure on both equations to get the same answer.

25 Michael: They're both giving the right answer, but they're different equations.

26 Heber: Yeah, we even eval [clears throat], we even evaluated at a different [clears throat] at a different height, for the tank. We evaluated it at one. We took their information at, er we just did another definite integral with theirs, and we went from negative five to negative four with um

27 Michael: Um hmm. [Janet Um hmm] Which is from there [points to the bottom of the tank] to there [points to the first horizontal line up].

28 Heber: We'll call that □□□□. [Justin moans] We did that with theirs and we did the same thing with ours, we went from zero to one, um □□□□ and we got the same exact answer for that so their equation models same, the same exact volume that ours does, uh, which doesn't completely make sense to me, but I'm sure someone understands it.

29 Michael: Alright. So that that's what I'm trying to resolve. Now, what I, what I wanna ss, uhh, think about is if we take an indefinite integral, of the equation that they had, no? That's what we did here, we took an indefinite integral and without an interval we simply got volume as a function of *h*. So if we do the same thing with their equation and take volume as a function of *h* Now, let's analyze, if we plug in an *h* value of one into our equation it will return this volume down there, is that making any sense?

30 Timbre: It seems like the same equation.

31 Heber: Oh, I get what you're saying.

32 John: I'm [sighs]

And while Heber expresses confusion in line 28, he finally indicates that he understands what Michael means by mathematical difference in line 31. Nevertheless, Timbre, in line 30, and probably Justin and John, in lines 28 and 32, as evidenced by their moan and frustrated sigh, have still not been able to interpret Michael's meaning for mathematical difference. Therefore, what constitutes a mathematical difference is still being negotiated and this qualifies as a quasi sociomathematical norm.

Here we point out, that Michael's quasi sociomathematical norm is at its budding stage. In the previous examples, the expectations of what constitutes a good explanation were first introduced in the first week of class. So those expections had gone through a longer negotiation prior to their invocation on this day in class. Michael's expectation, on the other hand, had only been considered for about 10 minutes. Therefore, the lack of agreement among students is not surprising and should be noted before one makes generalizations about the relative strength or weakness of the agreement within the negotiation process

The Role of Authority

As we were studying both the introduction and negotiation of sociomathematical norms, and the authority with which one introduces sociomathematical norms, we noticed that authority was playing two roles. Authority was needed both to introduce and to legitimize a sociomathematical norm.

Michael's initial statement in line 20, that "they both model something different," did not supply enough mathematical information for the class to build a mathematical interpretation of the expectation, nor to judge whether it was legitimate. When Michael began to explain his expectation he made explicit how he wanted to compare the two equations, and offered legitimization of the expectation through mathematical authority. Although Michael held granted authority to present his ideas to the class and used it to introduce the expectation, it was Michael's mathematical authority that legitimized his expectation through his mathematical argument, and allowed Heber to interpret the expectation.

Michael's introduction of the quasi sociomathematical norm of what constitutes a mathematical difference led to over ten minutes of explicit interpretation and negotiation among the participants. We believe that there are three contributing factors to the richness and depth of the interpretation and negotiation process in this case. First, the expectation was explicitly stated. Second, the quasi sociomathematical norm was introduced by a student. Third, the quasi sociomathematical norm was legitimized through mathematical authority.

On the other hand, in the negotiation of the quasi sociomathematical norm that a good explanation connects the model, context and equation, no member of the class ever directly stated the expectation. Instead students and instructors put forth mathematical explanations some which were accepted and some which were questioned or extended by members of the class or the instructors. When the explanations were accepted, they contributed to the agreement of the expectation and further contributed to the expectation becoming normative. However, when the explanations were not accepted, they contributed to the ongoing negotiation process of interpretation and agreement. While mathematical arguments were presented as a part of mathematical explanations (as in lines 1, 5, 11, 12, and 16), these arguments were not used to introduce or legitimize the quasi sociomathematical norm. Instead if any authority was used at all it was hierarchal authority in both its forms.

In fact, we saw very little evidence that this norm was being explicitly negotiated. What we saw, was that each time an explanation was made, either it was accepted implicitly by the class, or someone, often one of the instructors, offered a clarifying remark or question. Over time, with many explanations being offered there was ample opportunity for students to interpret the expectation which likely led to the fairly strong agreement that we witnessed among the participants for what constituted an acceptable explanation. However, the negotiation process was implicit. The lack of both explicit statement of the expectation and the use of hierarchal authority to legitimize the quasi sociomathematical norm could have very easily led to lack of agreement among the participants of the course.

Conclusions

We were able to identify the sociomathematical norm that *a good explanation of a new procedure or model requires a mathematical argument or mathematical authority*. In addition we identified three quasi sociomathematical norms. First, that a good explanation connects the model, context, and equation. Second, that *a different model creates different mathematics*, and third, that *two equations are different if they act differently on a common model*.

We found that both sociomathematical norms and quasi sociomathematical norms in this setting were introduced with hierarchal authorities whether the expectation was introduced implicitly or explicitly. We also found, in the case of the quasi sociomathematical norms that were introduced by Michael, a student in the class, mathematical authority played a strong role in the negotiation process including interpretation, and legitimization. And in fact, in the quasi sociomathematical norm that was introduced implicitly, authority did not play an observable role in the negotiation process and in fact, the negotiation process was implicit. We cannot tell from our data whether the negotiation process would have been more likely to be explicit had the expectation been made explicit or if the expectation had been made by a student. However, we did see a strong and explicit negotiation process in the case where the expectation was made explicit by a student and legitimized with mathematical authority.

In our first study on authority (Gerson & Bateman, 2010), we found that "in a mathematics class, it is those authorities that legitimize claims while also providing convincing evidence that have the highest potential to affect a student's mathematical understanding and mathematical autonomy" (p. 206). We believe that this is also true for the negotiation process of quasi sociomathematical norms. That is, that mathematical authority has the highest potential to allow for rich negotiation leading to strong agreement among the participants.

When instructors introduce sociomathematical norms, as in the study by Levenson, Tirosh, and Tsamir (2005), their hierarchal authority may legitimize the sociomathematical norm, but does not necessarily encourage students to interpret and come to agreement. Therefore students may be more likely to accept the norm before they agree on its mathematical meaning. Therefore, if teachers introduce a sociomathematical norm, they should be aware of the potentially obstructive role their hierarchal authority may play in the negotiation of that norm. The exercise of mathematical authority, through mathematical argumentation, by the instructor and students may influence the extent to which sociomathematical norms are truly agreed upon by both instructor and students.

Hierarchal authority also played a role in negotiating and legitimizing the sociomathematical norm of what constitutes a good explanation. But in this case, the norm was not explicitly negotiated. We believe that there were two likely causes. First, the expectation was never explicitly stated. And second, the hierarchal authority was only legitimized by the bearer's position in the class and therefore did not necessarily carry any mathematical weight through mathematical argument. In addition, we believe that quasi sociomathematical norms are best introduced explicitly so that the negotiation process can also be made explicit. Ideally a sociomathematical norm would be explicitly introduced by a student or instructor, through a hierarchal authority, and then legitimized by a mathematical argument, through mathematical authority. Additionally, if it is introduced by a teacher, we suggest that the teacher should find ways to introduce mathematical authority so that the quasi sociomathematical norm is not accepted without negotiation due to the teacher's institutional authority.

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